Dividend protection at a price: lessons from endowment warrants

Christine Brown and Kevin Davis*

Department of Finance, The University of Melbourne, Victoria, 3010. Australia.
Phone: (61) 3 83445308. Fax: (61) 3 83446914. Email: christine.brown@unimelb.edu.au

Abstract

The development of long term equity warrant markets has been hampered by the difficulty of providing dividend protection to investors in an easily understood form. A recent Australian innovation, the Endowment Warrant, is a long-term call option offering a form of dividend protection. Its simple structure has made it popular with retail investors. However, valuation of the option is complex, since it is a long term option with a stochastic strike price. This paper reviews what is meant by dividend protection and demonstrates that endowment warrants provide only a form of partial dividend protection. Examining the cost of incomplete dividend protection to investors indicates that the warrants issued have traded at prices significantly in excess of their fair value. Some possible causes of this divergence, and lessons for the construction of new securities are discussed.

The adjustment of contract terms required to provide dividend protection to investors in option contracts on dividend paying stocks has been known for three decades (Merton [1973]). Despite that, and surprisingly given the explosive growth of financially engineered products, there has been very little development of longer term equity option or warrant contracts providing dividend protection. One explanation for this lies in the difficulty of packaging a dividend protection feature into a warrant product in a form which can be easily understood by, and marketed to, the general investment community.

Long dated call options, known as Endowment Warrants, which were introduced to the Australian stock market in February 1996, appear to have overcome the problem of providing a simple form of dividend protection and have proved popular with retail investors, partly for this reason. For a price generally set between one and two thirds of the share price, the investor purchases an endowment warrant with an outstanding amount (the amount owed on the optional delayed share purchase) set by the issuer. This outstanding amount grows at the risk-free interest rate and is reduced periodically by application of each dividend paid on the stock to the outstanding amount. In effect, the endowment warrant is a long-dated call option with a varying exercise price given by the terminal date outstanding amount. The exact nature of the variation of the exercise price is discussed in Section 1 where a valuation model for the case of known dividends is presented.

Holders of unprotected call options have a claim only on the capital gains portion of the total stock return. In contrast endowment warrant holders have a claim on both the capital gains and the dividends, because the dividends that are paid on the stock are used to reduce the ultimate exercise price. However the form of the adjustment for dividends that is made to the endowment warrant contract does not *fully* protect the holder of the endowment warrant, as we demonstrate in Section 1. We show that the endowment warrant price decreases with increasing dividends, as a result of the fact that the dividend protection is incomplete.

Because endowment warrants provide incomplete dividend protection, the valuation model proposed by Hoang, Powell and Shi (HPS) [1999] is incorrect. In Section 2 we highlight the pricing differences between our model and that of HPS, and the implication for their conclusion that endowment warrants were fairly priced on issue. In Section 3 we develop a simulation model to value endowment warrants when interest rates and dividends are stochastic. Section 4 discusses some of the explanations for why endowment warrants may trade at prices in excess of fair value and provides conclusions.

1. Endowment Warrant Valuation in the Case of Known Dividends

Endowment warrants are exchange traded, third party issued, European long dated call options with a stochastic exercise price on listed stock of major Australian companies[1]. They were issued at prices between around 30 percent to 65 percent of the current share price of the underlying stock, and the initial outstanding amount (sometimes loosely referred to as the initial exercise price), has varied from between 95 to 60 percent of the current share price.

To illustrate, the initial issue price of an endowment warrant on a stock trading at \$10.00 might be set at \$5.00 with an outstanding amount of \$6.50. This outstanding amount is adjusted over time in the following way: it increases daily by application of a benchmark interest rate (reset at regular intervals) to the outstanding amount, and decreases stepwise (semi annually) by the amount of dividends (and other distributions) paid on the stock.[2]

Over time, therefore, the outstanding amount is expected to decline in a sawtooth pattern (provided dividends on the stock exceed interest accruing on the outstanding amount), and is initially set so that its expected termination date value is zero, provided that the dividend growth rate and interest rate forecasts of the issuer are realised. The termination date is set at around 10.5 years from the issue date. An endowment warrant is therefore a call option over the underlying stock with a stochastic exercise price (due to stochastic dividends and interest rates) and stochastic exercise date (because the outstanding amount may reach zero before the termination date). The exercise date of the warrant occurs 30 days after the earlier of (i) the termination date and (ii) the date on which a dividend payment is sufficient to reduce the outstanding amount to zero.[3]

We first provide a simple derivation of the appropriate pricing model in the special case where dividends are known and the interest rate is constant, before relaxing these assumptions in Section 3. Exhibit 1 provides a summary of notation.

Note that the endowment warrant is a European option and thus the terminal date (T) value of the outstanding amount process (K_T) is the option exercise price which is of relevance. That value is the outcome of the process K_t which commences at K_0, grows continuously at rate r with six monthly discrete drops equal to the dividend amount paid at those dates. K_T can also be related to the outcome of an alternative process Z_t which commences at $Z_0 = K_0$. The process Z_t grows continuously at rate r such that $Z_T = K_0e^{rT}$. The relationship between Z_T and K_T is given by:

$K_T = Z_T - D$

where D is the future (date T) value of all dividends paid between 0 and T and reinvested at the interest rate r. In the case of a fixed interest rate r, Z_T is known at date 0 and the endowment warrant exercise price is thus equal to this known exercise price reduced by the future value of dividends paid on the stock. This deduction is also known at date 0 in the case of known dividends and thus the exercise price is non-stochastic.

How should this simplified, non-stochastic exercise price, endowment warrant which is equivalent to a standard call option on a dividend paying stock with exercise price of $Z_T - D$ be valued? Let the date 0 present value of dividends to be paid between 0 and T be represented by D*. It is well known that the Black and Scholes [1973] and Merton [1973] European option pricing formula denoted by $C(S,X, \sigma, r, T)$ is modified to $C(S\text{-}D^*, X, \sigma, r, T)$ for an option with exercise price X on a stock paying known dividends with present value D*. In this case, the exercise price is given by $K_T = Z_T - D = K_0e^{rT} - D$ and the relationship between the future and present value of dividends is given by $D^* = De^{-rT}$. Hence, the appropriate pricing formula can be represented by $C_0 (S_0 - D^*, Z_T- D, \sigma, r, T)$. Using the standard modified Black-Scholes formula and substituting for Z_T and D gives

$$C_0 = (S_0 - D^*).\ N(d_1) - e^{-rT}(Z_T - D)N(d_2) \quad (1)$$

$$= (S_0 - D^*).\ N(d_1) - (K_0 - D^*)N(d_2) \quad (2)$$

where:

$$d_1 = [\ln((S_0\text{-}D^*)/(Z_T\text{-}D)) + (r + \sigma^2 /2)T] / (\sigma\sqrt{T})$$

$$= [\ln((S_0\text{-}D^*)/(K_0\text{-}D^*)) + \sigma^2 /2T] / (\sigma\sqrt{T}) \quad (3)$$

and $$d_2 = d_1 - \sigma\sqrt{T}. \quad (4)$$

We refer to Equation (2) as the EW model, which would apply to endowment warrants in the special case of known dividends and fixed interest rate. Hoang, Powell and Shi (HPS) [1999] published a different model, for the case of constant interest rates. Their model assumes that the dividend protection is complete, resulting in the following valuation.

$$C_0 = S_0 . N(\delta_1) - K_0 . N(\delta_2) \tag{5}$$

where:

$$\delta_1 = [\ln(S_0/K_0) + (\sigma^2 /2)T] / (\sigma\sqrt{T}) \tag{6}$$

$$\delta_2 = \delta_1 - \sigma\sqrt{T} \tag{7}$$

HPS argue that the endowment warrant can be thought of as equivalent to a dividend protected call option with an exponentially increasing strike price. It is true that the form of 'dividend protection' given in Equation (4) of their paper results in the same payoff for an endowment warrant and a 'dividend protected' call option. However, this form of dividend protection created by subtracting the dividend from the outstanding amount, or equivalently subtracting the future value of dividends from the terminal date outstanding amount, is incomplete. Assuming complete dividend protection applies (which they do) implies that the warrant can be valued *as though* the stock does not pay dividends, which results in Equation (5), a simple model with no dividend terms in it. However this assumption is not correct for the strike price adjustment of the endowment warrant. Indeed Merton [1973] shows that only when dividends are reinvested in the stock is full dividend protection afforded.

Merton [1973] and Geske, Roll and Shastri [1983] demonstrate that the form of adjustment where the strike price is reduced by the size of the cash dividend at the ex-dividend date (called OTC protection by Geske, Roll and Shastri) results in imperfect dividend protection. Geske, Roll and Shastri show that the size of the loss to a call option holder as a result of imperfect dividend protection is small for short maturity options, and the loss is an increasing function of the size of the dividends, the interest rate and the maturity date. They compare the ex dividend date value of an OTC protected call option on a stock with cum dividend price of S, dividend payment of d, and pre adjustment strike price of X with the corresponding value of a call option on an otherwise equivalent zero dividend stock. To a first approximation, the value of the OTC protected call is d/S per cent lower.

Similarly, the endowment warrant adjustment of reducing the exercise price by the future value of the dividend (or equivalently reducing the outstanding amount by the actual value of the dividend) does not provide full dividend protection. This is illustrated by differentiating Equation (2) with respect to D* and simplifying, resulting in:

$$\frac{\partial C}{\partial D^*} = N(d_2) - N(d_1) < 0 , \tag{8}$$

which indicates that dividend protection is less than complete. Thus the HPS [1999] formula will overvalue endowment warrants, except in the trivial case of zero dividends. Exhibit 2 provides an illustration of the errors involved in the HPS model for the case considered in this section and where the present value of dividends equals the outstanding amount, demonstrating that the size of the error increases as the outstanding amount (relative to the stock price) increases.

Equation (8) shows that the endowment warrant price increases as the present value of dividends decreases. In other words, a lower dividend payout makes the warrant more valuable (contrary, we suspect, to the perceptions of most retail investors in the product). Exhibit 3 (where the present value of dividends is scaled by the outstanding amount) illustrates and also compares prices from the HPS [1999] model and the EW model (equation (2)). When dividends are relatively small, towards the left of the horizontal axis in Exhibit 3, the cost to the warrant holder of this imperfect dividend protection becomes less significant, and thus prices given by the HPS model and the EW model are closer. The EW valuation line is almost horizontal when the ratio of present value of dividends to the outstanding amount is close to 1, showing that in this region the warrant valuation is relatively insensitive to changes in the present value of dividends. As expected in the case where the ratio is 1, such that the terminal date strike price is known to be zero, the endowment warrant value equals the difference between the stock price and the outstanding amount.

More generally, as shown by Merton [1973], there is no simple dividend-related adjustment to the strike price alone which can provide full dividend protection. The endowment warrant exercise price adjustment is a simple one which is easy for retail investors to understand, but comes at the expense of incomplete dividend protection. The effect of interest rate and dividend uncertainty on endowment warrant valuation is considered in the following section.

3. Dividend Uncertainty and Endowment Warrant Valuation

In this section we develop a simulation model to value endowment warrants and compare the prices from the simulation model, with prices from the EW model and the HPS model. In order to compare the results from our simulation model with the prices produced in the HPS paper we use the same data (including observed market prices) as in Exhibit 4 of their study, where valuations are performed at 1 October 1998.

A practical valuation model for endowment warrants must take account of the variability in the short term interest rate applied to the outstanding amount, uncertainty in dividends which are paid semi-annually, and early termination of the endowment warrant when the outstanding amount goes to zero. Clearly, the Black-Scholes-Merton style model of the previous section for the case of known dividends cannot deal with these features of the endowment warrant. Complete treatment of the impact of dividend and interest rate uncertainty requires the use of numerical techniques. In this section we present the results of Monte Carlo simulation under the assumption of geometric Brownian motion for stock prices, using the Cox, Ingersoll and Ross [1985] model for interest rates and uncertainty introduced into the dividend amount through an assumption of a constant dividend yield.

The stock price path is simulated as Geometric Brownian Motion using Monte Carlo simulation with one-month time steps, assuming that a dividend is paid at each sixth step. The dividend amount is calculated as a constant dividend yield multiplied by the stock price at the previous step.[4] The assumed dividend yield is derived by setting the initial dividend equal to the most recently paid dividend for each stock. We assume an ex-dividend drop-off in the share price equal to the cash dividend.

Interest rates are simulated using the Cox, Ingersoll, Ross [1985] model with the short term rate set to 4.88% and the long term rate set to 5.02%[5], with adjustment rate of 25%. A correlation of zero is assumed between movements in the stock price and short term interest rates.

To produce prices using the analytical EW model, given by Equation (2), we calculate the present value of dividends to substitute in that model as follows. First we take the most recent dividend prior to 1 October 1998, and calculate a dividend yield using the dividend announcement date stock price. Assuming the dividend yield is constant over the life of the warrant we then calculate the present value of dividends from 1 October 1998 until the maturity date of the endowment warrant.[6]

Given the valuation date of October 1998, the remaining time to maturity is 7.75 years. The proportion of simulations in which the outstanding amount reaches zero prior to 7.75 years is recorded and the effect of earlier termination incorporated into the warrant valuation.

The results for the analytical model (EW), the simulated warrant prices (SIM) and the HPS [1999] prices are presented in Exhibit 4. For actual warrant prices observed on 1 October 1998, the EW and simulation models on average underprice relative to observed market prices, whereas the HPS model on average overprices.

It is interesting to compare the EW model prices with those produced by our simulation model. The EW model values the warrant with a fixed maturity date because the dividends are known in advance. In contrast, the simulation model allows for early termination of the warrant contract should the outstanding amount go to zero before the contractual maturity date. Both models have the same assumed dividend yield and present value of future dividends. Simulation model prices are close to those produced by the EW model, except in three cases.

The stocks for which there is a large difference between the EW model and the simulation model are BOR, CSR and SGW. On 1 October 1998 each of these warrants had outstanding amounts that were greater than the stock price (despite being initially issued with outstanding amounts around 80 percent of the then stock price). These warrants were significantly out-of-the-money and in each of these cases a large percentage of the simulation price paths have terminal stock prices where the warrant finishes out-of-the-money, as column 5 of Exhibit 4 illustrates. In these cases the simulation model produces prices that are considerably lower than those produced by the EW model.[7]

Underlying this difference is the path dependency of actual dividend amounts in the simulation model, where the constant dividend yield means that dividends are positively correlated with the average stock price over the path. Because dividend protection is incomplete, high dividend outcomes reduce the value of the warrant. While low dividends conversely would tend to increase the value of the warrant, this offsetting effect does not occur for those paths where the warrant finishes out of the money and thus has zero value. In contrast, the EW model, while assuming the same average dividend yield and dividend amount, implicitly assumes a zero correlation between the dividend amount and the stock price path. For those warrants which are well in the money and where the simulation model has no cases of price paths finishing out of the money, the resulting linear payoff schedule of terminal date warrant values means that the path dependency of dividends does not affect the current warrant price and EW and simulation results are thus relatively close.

We also observe that the simulation model produces estimates for the warrant value that are on average below market prices. This is in contrast to the HPS formula which produces prices which are on average higher than the market prices. The greater values resulting from the HPS formula reflects the (inappropriate) assumption of full dividend protection. These results suggest that endowment warrants have traded at prices in excess of fair value, some explanations for which are considered below.

4. Discussion and conclusions

The dividend protection provided to investors in the structure of the endowment warrant is incomplete. We have provided a simple analytical model for the case of known dividends which shows that the value of the endowment warrant increases as dividends decrease. This result is consistent with the findings of Merton [1973] and Geske, Roll and Shastri [1983] and arises because investors receive a higher capital gain when dividends are lower, which more than compensates for the higher strike price at the maturity date of the warrant. We show that the magnitude of the pricing error induced by assuming that the dividend protection is complete can be substantial.

Using a simulation model that incorporates stochastic interest rates and dividends, we produce prices for endowment warrants and compare them with both market prices and the prices produced using the HPS [1999] model for particular warrant prices observed on 1 October 1998. These results show that prices produced by the simulation model, and the analytical model for the case of known dividends, are lower on average than market prices, and lower on average than the prices produced by the HPS model.

Several possible explanations can be advanced for such apparent mispricing. Endowment warrants may provide tax arbitrage strategies for some (international) investors – although Brown and Davis [1997] demonstrate that the warrants are tax disadvantaged for Australian investors. Because endowment warrants are a partial substitute for levered purchases of stock and are available in small parcel size and involve wholesale market interest rates, they provide liquidity benefits and overcome borrowing market imperfections for retail investors looking for leveraged positions. In addition, legal constraints on institutional investors (such as pension funds) taking leveraged positions may be effectively evaded by investment in endowment warrants.

The most important contribution of this paper however lies in enhancing our understanding of the appropriate design and pricing of warrant products which provide some measure of dividend protection to investors. As demonstrated, the differences in fair value between longer term warrants which provide partial and full dividend protection can be very significant, indicating the importance of full appreciation of the extent of dividend protection provided by innovative products.

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Exhibit 1: Notation

C_t	price of warrant at time t
S_t	price of underlying stock at time t
σ	volatility of the underlying stock
K_t	outstanding amount at time t
T	expected maturity date of warrant
Z_t	Date t value of the initial outstanding amount compounded at the interest rate
d_t	dividend at date t
D_t	date t value of accumulated dividends (reinvested at the interest rate).
D*	present value of dividends over the expected life of the warrant
D	Future value (at date T) of dividends
r	risk-free interest rate

Exhibit 2: An illustration of the cost of incomplete dividend protection

This table gives the percentage overpricing of the HPS model for the endowment warrant with maturity of ten years, constant interest rate of 7.5%, where it is assumed that the present value of the known future dividends equals the outstanding amount, such that the terminal date exercise price is zero and certainty of exercise exists.

Volatility (%)	**Outstanding Amount as Percentage of Stock Price**			
	50%	**60%**	**70%**	**80%**
10	0.22	1.36	5.68	19.87
20	5.99	14.10	30.71	68.30
30	17.06	32.27	59.95	118.30

Exhibit 3

Endowment Warrant valuation using the HPS model (equation 5) and the Endowment Warrant model (equation 2) as a function of the ratio of dividends to outstanding amount

(S_0= 10, Outstanding Amount = 7, σ = 0.2, T = 10)

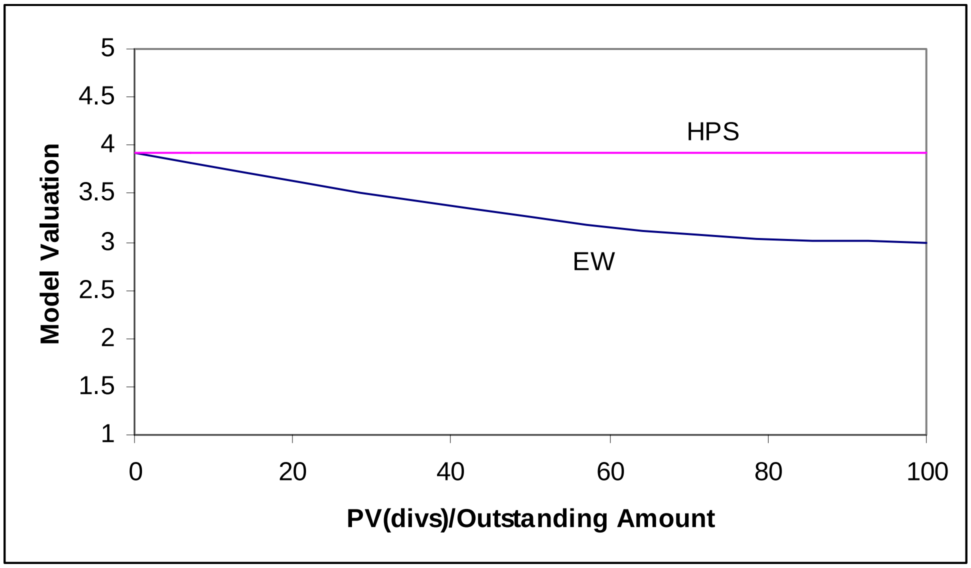

Exhibit 4
Endowment Warrant Simulation Results

This table shows the estimated endowment warrant prices at 1 October 1998 from the EW model (equation 2) and from Monte Carlo simulations (SIM) assuming dividends are paid discretely every six months and interest rates follow a CIR process. Zero correlation is assumed between the stock price changes and interest rate changes. The simulation results are based on 2000 simulations, using the antithetic variable technique. The prices are produced using the volatility assumptions contained in Exhibit 4 of the HPS (1999) paper. The column labelled %OTM shows the percentage of paths from the simulation where the warrant finished out of the money. These are prices for the Credit Suisse Endowment warrants where the dividends do not have tax credits attached to them (cash only is applied against the outstanding amount). The market prices are the mid-point of the bid-ask prices on 1 October 1998. BRY and NCM warrants subsequently ceased trading and have not been included in the tabulated results. The root mean squared error of the simulation is reported in column 4.

							Difference with Traded Price		
Code	**EW**	**SIM**	**RMSE**	**%OTM**	**HPS**	**Market**	**EW**	**SIM**	**HPS**
ANZ	4.04	4.12	0.09	0.00	4.51	4.19	-0.15	-0.07	0.32
BOR	0.33	0.16	0.02	81.00	0.49	0.55	-0.22	-0.39	-0.06
BIL	27.53	27.63	0.29	0.00	27.54	27.84	-0.31	-0.21	-0.30
CML	4.09	4.12	0.05	0.00	4.16	4.17	-0.08	-0.05	-0.01
CBA	11.19	11.36	0.10	0.00	11.32	11.43	-0.24	-0.07	-0.11
CSR	0.63	0.17	0.02	81.00	0.74	0.61	0.02	-0.44	0.13
FBG	2.08	2.04	0.04	0.00	2.12	2.16	-0.08	-0.12	-0.04
GIO	2.51	2.36	0.06	0.00	2.75	2.47	0.04	-0.11	0.28
LNN	1.76	1.60	0.04	0.00	1.98	1.89	-0.13	-0.29	0.09
MBL	7.64	7.92	0.18	0.00	8.16	7.97	-0.33	-0.05	0.19
NAB	9.79	10.07	0.12	0.00	10.17	10.10	-0.31	-0.03	0.07
STO	1.48	1.39	0.04	0.00	1.85	1.82	-0.34	-0.43	0.03
SGW	0.67	0.29	0.04	82.00	1.22	1.27	-0.6	-0.98	-0.05
TEL	2.29	1.82	0.08	1.00	2.66	1.90	0.39	-0.08	0.76
WOW	4.11	4.08	0.04	0.00	4.12	4.06	0.05	0.02	0.06
						Average difference	**-0.15**	**-0.22**	**0.07**

ENDNOTES

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[1] A specified number of warrants on each company was created by the issuers, which were sold to the public in the initial offering, as a tap issue at a price which can vary over time, through an intermediary (Challenger International). Reflecting the popularity with retail investors (for whom they are a possible alternative to margin lending) there have been several series of endowment warrants created on the same stocks subsequently.

[2] The interest rate applied on warrants issued by one of the issuers, Macquarie Bank, is the 90 day bank bill rate plus 300 basis points, while that on warrants issued by Credit Suisse Financial Products is the 180 day bank bill rate. The dividend amount credited against the outstanding amount is the cash value of the dividend in the latter case, and the cash value plus the value of imputation tax credits of dividends in the former case. Dividends are typically paid semi annually in Australia and may carry tax credits for the investor. In this paper the warrants studied are those issued by Credit Suisse where the dividends do not carry the tax credits.

[3] If the dividend payment exceeds the outstanding amount, the warrant holder is entitled to the excess.

[4] Simulations involving a dividend smoothing assumption (see Lintner [1956]) produced virtually identical results, reflecting the minor impact of smoothing on the present value of total dividends paid over such a long time horizon.

[5] These were the market rates on 1 October 1998.

[6] We use Se^{-qT} to approximate the stock net of the present value of dividends, where q is the dividend yield. Then the present value of dividends can be approximated by $S(1-e^{-qT})$. Checks were run comparing the calculated present value of dividends against the present value of dividends as an output from the simulation. The results are similar.

[7] We run robustness checks on our simulation model and find prices produced by using the same simulation model for stocks and dividends, but holding the interest rate constant at the initial short term rate, produces prices where the average percentage difference in price between the two models is 2%.