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OPTIONS IN MUTUALLY EXCLUSIVE PROJECTS

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ABSTRACT:

Standard techniques advocated for choosing between mutually exclusive projects of unequal life make an implicit assumption of continued project replication. While intuitively appealing, those techniques ignore the fact that project replication is one outcome of a *repeated choice* situation, and may not be the optimal outcome once stochastic features of the environment are taken into account. In essence, standard techniques ignore the real options relating to subsequent choices to be made which are inherent in each decision By means of a simple example, involving a simple specification of interest rate uncertainty, it is demonstrated that the standard techniques can lead to error in a stochastic environment. Because of the idiosyncratic characteristics of project comparisons and the compound nature of the options involved, neat analytical solutions and techniques are not available to replace the elegant, but inadequate, textbook models. Financial managers need to model each choice on a case by case basis, appropriately identifying the key drivers of NPV and specifying the stochastic environment pertaining to each.

NPV analysis does not allow for management's dynamic reaction to uncertainty in the operating environment. Rather the traditional DCF approach to capital budgeting, takes risk or uncertainty into account either by adjusting risky cash flows to certainty equivalents and discounting at the risk free rate, or by discounting risky cash flows at a risk-adjusted rate. Recognizing that an investment opportunity is like a financial call option clarifies the role that uncertainty and information play in project evaluation.

The development of the real options approach to capital budgeting has transformed the way in which capital projects should be evaluated. Real options which have been identified and valued in the literature include the option to defer investment (McDonald and Siegel (1986) and Ingersoll and Ross (1992)); the option to alter the operating scale of the project (Trigeorgis and Mason (1987), Pindyck (1988), Brennan and Schwartz (1985) and the option to abandon (Myers and Majd (1990)).

However, the full effects of those developments have not found their way through to either practice or the conventional wisdom (as exemplified in most textbooks). Indeed many commonly advocated procedures implicitly ignore the existence of options which might realistically be thought to be important in practice, and the objective of this paper is to outline the errors which can occur in one such case.

The situation examined here is the one in which an organization is faced with making a (mutually exclusive) choice between two projects with unequal lives. It is well known that a simple comparison of project NPV's is inappropriate in these circumstances. Consequently a number of approaches have been advocated, each of which involves assumptions aimed at enabling a comparison of like with like. These approaches include[1]:

* Equivalent Annual Value, in which the annuity equivalent of the NPV is calculated and an assumption made that by continuous replication of each project a corresponding perpetuity can be created
* Constant Chain of Replacement, in which it is assumed that projects are replicated infinitely and the NPV of that infinite chain of NPVs calculated
* Least Common Multiple, in which it is assumed that each project is replicated a sufficient number of times such that a common end point is achieved, and the NPV then determined.

All of these approaches make a common assumption that the subsequent project chosen is the same as the current project. Thus, if it is optimal to choose project A, which has an n year life, today it is assumed that it will remain optimal to

[1] See, for example, Brearley and Myers (1991, chapter 6), Copeland and Weston (1988, chapter 3), Van Horne, Wachowicz, Davis and Lawriwsky (1995, chapter 11), Pierson, Bird, Brown and Howard (1995, chapter 18)

again choose project A in n years time. This is a strong assumption. Not only is there uncertainty about future technology and thus about what, as yet unknown, alternative projects might compete with A, there is no guarantee that parameters such as interest rates, capital outlay costs, cash inflows inherent in the NPV calculation will remain unchanged over time. It is possible that at date n, these parameters may have changed in such a way as to now make project B the optimal choice.

The implicit assumption of the standard approaches is that the same decision will be optimal at all future dates when a decision is to be made. If so, assumption of continued or infinite replication is appropriate. However, if there is uncertainty about future NPVs, it cannot be assumed that optimal choice is replication. In the case of a multi-stage project the traditional approach would assume that management is passive. Such an approach does not take into account that with managerial flexibility a different decision may be made at a later stage in the project if market conditions change. Ingersoll and Ross (1992) have shown that with uncertain interest rates, projects have an attached option value, and furthermore that this option value can be substantial.

The issue we thus face is that projects of different lives give rise to subsequent choices (options) at different future dates. Do these options have different values and therefore possibly affect choice?

Fundamental to our argument is the recognition that in choosing between two projects A and B with lives of n and m years respectively, we are choosing between two compound projects. The first is to choose A with a life of n years and receive an option after n years to choose again between A and B. The second is to choose B with a life of m years and receive an option after m years to choose again between A and B. The optimal choice today should thus compare the NPV of a "one-shot" choice of A plus the option after n years with the NPV of a "one-shot" choice of B plus the option after m years. The issue which thus needs to be considered is the nature of those options and their valuation - which is, unfortunately, complex because each option involves further subsequent choices.

To illustrate the general arguments of this paper we utilize a simple example as outlined below.

The Standard Approach: An Example

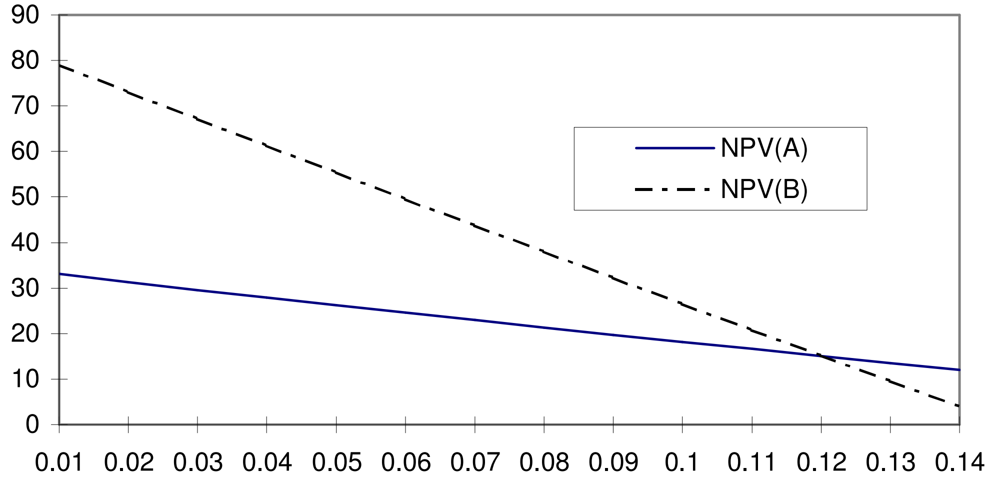
We consider a company which has a choice between two projects of finite, different, lives, for example a Transport company with a choice between 2 trucks, one with a three year life, the other with a five year life. Rather than set out cash flow patterns for each project we make use of the NPV schedule which relates the NPV of each project to the discount rate used.

Project A: 3 years, $NPV_A = 35+80i^{1.4}-200i$

Project B: 5 years, $NPV_B = 85+150i^2 -600i$

Figure 1 provides an illustration of the NPV relationships for the two projects

Figure 1
"One-shot" NPVs



A standard approach to this problem such as the constant chain of replacement approach would involve a comparison of the NPV from an infinite repetition of A given by

$$NPV(A, \text{infinitely}) = NPV_A^* = NPV_A/[1-(1+i)^{-3}]$$

with that of an infinite repetition of B given by

$$NPV(B, \text{infinitely}) = NPV_B^* = NPV_B/[1-(1+i)^{-5}]$$

These NPV relationships are shown in Figure 2 and the underlying data in Table 1.

Figure 2: Infinite Replication NPV's

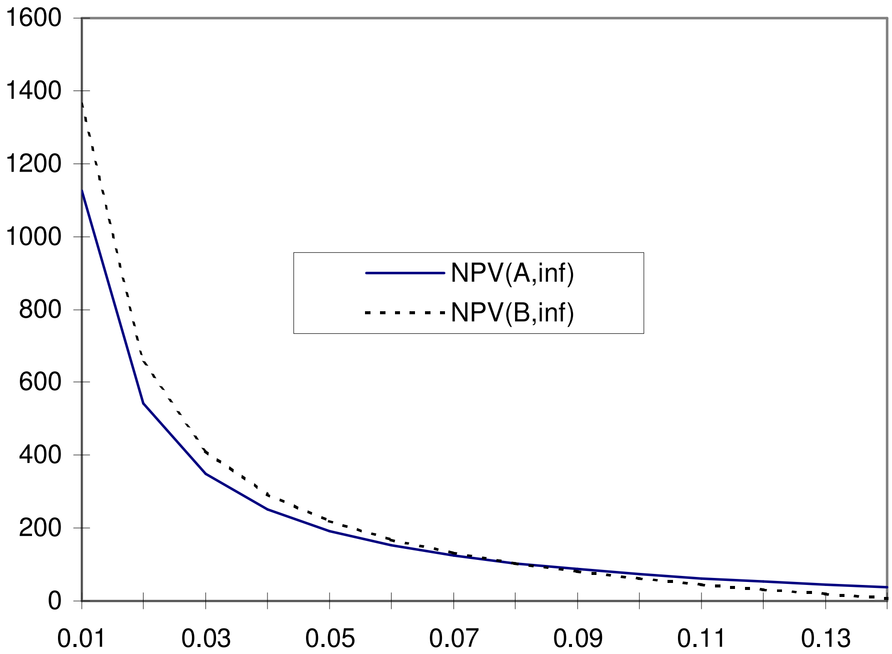


Table1: NPV relationships					
i	NPV(A)	NPV(B)	NPV(A,infinitely)	NPV(B,infinitely)	
0.01	33.1268	79.015	1126.38	1363.39	
0.02	31.3346	73.06	543.27	652.15	
0.03	29.5903	67.135	348.70	413.10	
0.04	27.883	61.24	251.19	292.06	
0.05	26.2068	55.375	192.47	218.20	
0.06	24.5578	49.54	153.12	167.91	
0.07	22.933	43.735	124.84	131.08	
0.08	21.3303	37.96	103.46	102.64	
0.09	19.7481	32.215	86.68	79.79	
0.1	18.1849	26.5	73.12	60.85	
0.11	16.6395	20.815	61.90	44.73	
0.12	15.111	15.16	52.43	30.73	
0.13	13.5984	9.535	44.30	18.35	

To consider the standard approach, note that if the interest rate is currently 8%, $NPV_A^* = 103.461$, $NPV_B^* = 102.641$ and the optimal choice if infinite replication is assumed is to choose A.

Allowing for Optionality

To demonstrate our fundamental point, we assume a simple source of uncertainty and examine how that impacts upon the optimal decision. Specifically, we assume that interest rates are initially at 8% and that a one-off equi probable permanent change to 7% or 9% can occur at the end of 3 years. We also assume risk neutrality, which is a simplifying assumption which enables us to compare expected future NPVs without the complications introduced by the uncertainty associated with the distribution of future NPVs.

In this simple world, the decision problem faced is a relatively simple one. If A is chosen now, the company will be faced with a choice between A and B in three years when there is an equal probability that the interest rate could be 7% or 9%. Because there is no further uncertainty beyond that date, the constant chain of replacement approach is an appropriate vehicle for making the choice at that date. Similarly, if B is chosen, the company is faced with a choice in 5 years time, when interest rates will be either 7% or 9% and again with no further uncertainty.

If the interest rate is 7% the optimal next choice is to choose B which if infinitely repeated has an NPV at that date of 131.07. If the interest rate is 9% the optimal next choice is to choose A which if infinitely repeated has an NPV at that date of 86.68.

If A is chosen now, the NPV from that decision is the NPV from A over the next three years plus the present value of the optimal choice in three years time, which is given by:

$$NPV = NPV_A + 0.5[NPV_A^*(07\%)/1.08^3 + NPV_B^*(09\%)/1.08^3]$$

$$= 21.33 + 0.5[131.07 + 86.68]/1.08^3 = \mathbf{107.76}$$

If B is chosen now

$$NPV = NPV_B + 0.5[NPV_A^*(0.7\%)/1.08^5 + NPV_B^*(09\%)/1.08^5]$$

$$= 37.96 + 0.5[131.07 + 86.68]/1.08^5 = \mathbf{112.06}$$

It can be seen that the optimal decision is now to choose B, so that recognition of optionality at subsequent dates has changed the optimal decision from A to B. In this example, under infinite replication ignoring optionality the company chooses the shorter term project, while if allowing for the option element it chooses the longer term project.

The General Problem

Our simple example is illustrative of a more general problem which warrants consideration by companies choosing between mutually exclusive projects (A and B for example) of different lives. In essence, the choice of project A with a life of n years creates value equal to the NPV of cash flows from the n year project plus the NPV of the option to choose between A and B after n years.

Choice of B with life of m years creates value equal to the NPV of cash flows from the m year project plus the NPV of the option to choose between A and B after m years.

Unfortunately, a solution to this problem is not a simple one. The difficulty is that the options in question are an infinite regress of compound options. The options in question are the right to choose between A (with a further option after further n years) and between B (with a further option after further m years). In our simple example, we avoided this difficulty by assuming that there was only a one-off possibility of interest rate change giving rise to a simple choice structure.

In the more general case, a solution requires some form of dynamic programming approach, together with the specification of the stochastic environment which gives rise to option value. Obvious sources of such option value could be interest rate volatility or cash flow uncertainty (such as might occur in terms of the supply price of the capital items in question). Unfortunately, no general decision rules are likely to be readily available, since each situation will require its own unique specification of the stochastic issues involved.

Implications for Practice

Textbook authors have generally noted that the assumption of project replication may not be appropriate in particular situations and that some specific assumption about future decision possibilities may therefore need to be made. Typically, the examples considered involve situations where replication will not occur and some assumptions might need to be made about the NPV opportunities available to the firm at the termination of the shorter lived project.

However, the problem at hand is more pervasive. Decision makers faced with mutually exclusive projects of unequal lives will often find themselves in a *repeated choice* situation. Project replication is special case of a repeated choice situation where the optimal decision at each choice node is unchanged from the previous decision.

Our simple example illustrates that even if a repeated choice situation is to occur, project replication may not be the outcome of optimal decision making. Consequently, the assumption of continued replication made in standard techniques is inappropriate and has the potential to lead to error. While exactly the same projects may be under consideration at each choice node, the economic environment may have changed in ways which make replication non optimal. The message flowing from this is that evaluation of mutually exclusive projects of unequal lives needs to involve a much more explicit analysis of the stochastic environment within which decisions are made. Key variables influencing the "one-shot" NPVs of each project need to be identified and their stochastic behavior modeled. Particularly important amongst these are likely to be interest (discount) rates, supply prices of capital goods, and product demand projections which influence the cash inflows from the projects. Further complications (not considered in this paper) arise when mutually exclusive projects have inherent abandonment or extension options. Unfortunately, the idiosyncratic nature of the stochastic issues and project options applicable to any real world project choice situation mean that no simple "textbook" decision rules can be advanced. Students and practitioners should be exposed to the standard techniques but need to be made aware of the inherent dangers of an unthinking assumption of automatic project replication.

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