The Cost of Capital of a Company Under An Imputation Tax System:
Some Clarifying Comments

Kevin Davis*
Colonial Mutual Professor of Finance
Department of Accounting and Finance
University of Melbourne
Parkville, Vic 3106
Australia
Ph (61) 3 9344 5098
Fax (61) 3 9344 6681
Email k.davis@ecoaccfin.unimelb.edu.au

Abstract: In the May 1994 issue of Accounting and Finance, Bob Officer proposes a new definition of the cost of capital, and illustrates “consistency” relationships between alternative cost of capital definitions and cash flow measures. This article clarifies some definitional issues involved in Officer’s presentation, explains the limitations on applying the consistency results Officer derives in that paper and in his 1981 paper (Officer, 1981), and argues that Officer’s proposed measure is not superior to the more traditional definition of the cost of capital.

*I would like to acknowledge the very helpful comments of Tim Brailsford and John Handley, without implicating them in responsibility for any remaining errors.
Introduction

The cost of capital of a firm is generally defined as the discount rate which makes the capitalised value of the firm’s expected future cash flows equal to the firm’s market value. It has long been known (Nantell and Carlson (1975), Officer (1981)) that numerous definitions of the cost of capital can be derived, based on differing treatments of company tax payments in the definition of the firm’s future cash flows. What is not so widely appreciated, however, is the fact that not all versions of the cost of capital definition are equally useful for financial decision making. Indeed, even if cash flows are measured consistently with the cost of capital definition chosen, some cost of capital definitions seem likely to lead the financial analyst into error in some circumstances. Since the cost of capital plays an important role in corporate valuations, capital budgeting, and capital structure determination, it is important that an appropriate definition (and hopefully one suitable for use in all areas of financial management) be adopted.

Officer (1994) has outlined relationships between alternative cost of capital measures and advocated one particular measure for use under Australia’s dividend imputation tax system. This paper challenges Officer’s assertion that his proposed approach is preferable to the more traditional approach. It demonstrates that while Officer’s measure can be used in making accept-reject decisions in capital budgeting, Officer’s measure is not superior to the traditional approach on this criteria and has no basis in finance theory. Moreover, uncritical adoption of some of the alternative cost of capital measures presented by Officer could lead analysts into some serious valuation errors. This paper points out that some of those alternatives cannot be correctly implemented in practice, and why attempts to implement them will lead to errors.

Some Terminology

Officer’s (1994) analysis of the cost of capital under imputation is largely premised on the observation that tax paid by Australian companies under the imputation system can be interpreted as, in part or whole, a prepayment of personal tax obligations of shareholders. Consequently, he adopts the terminology of defining that part of tax paid
by companies which is not a prepayment of personal tax as company tax. Suppose, for example, a company made tax payments of $36 (based on a statutory company tax rate of 36 per cent applied to profit of $100) and paid franked dividends of $64 which carry with them imputation tax credits of $36. If shareholders of the company subsequently claimed $20 of those tax credits against their income tax liability, Officer’s terminology involves defining the amount of company tax paid as $16, and treating the remaining $20 as a prepayment of investor (personal) tax.

In his analysis, Officer denotes the effective rate of tax paid by a company by T, where this is the ratio of tax actually paid by the company (ignoring whether it is to be interpreted as prepayment of personal tax or not) to the company’s profit before tax. This creates some problems of interpretation, because T may differ from the statutory company tax rate (currently 36%) for a number of reasons. In what follows, I assume that the effective rate of tax (T) and the statutory rate are equal, in order to avoid unnecessary complications for the exposition.

Officer’s analysis is based on redefining the term company tax rate to reflect the interpretation of company payments of tax as prepayments of shareholder personal tax. Thus, if γ is the proportion of the tax credits distributed by companies which is used to offset personal tax liabilities, his definition of the company tax rate is (1-γ)T. In the example used above, where only $20 of $36 of tax credits were utilised, γ = 20/36 = 0.55, and Officer’s definition of the company tax rate would be (1-γ)T = (16/36)(0.36) = 0.16.

---

1 The tax rate utilised in applications should be the effective tax rate, which can differ from the statutory rate because of the effect of accelerated depreciation allowances, accumulated losses, and foreign source income. See Van Horne, Davis, Nicol and Wright (1990, pp 281-2) for a detailed discussion.
2 More precisely, γ is the proportion of tax paid by companies which is used to offset personal tax liabilities, but Officer implicitly assumes a 100 per cent payout rate of franked dividends. Companies may adopt a suboptimal dividend policy (from a tax perspective, see Howard and Brown, 1992) and not pay sufficient franked dividends to clear their franking account balances.
When Officer refers to the cost of capital after company tax in an imputation tax system, he is referring to a calculation based on this non-standard definition of company tax.

It is worth stressing that Officer’s γ refers solely to some implicit value of franking credits. It is a measure of the reduction in direct personal tax payments which follow from the distribution of $1 of franking credits. It is completely unrelated to the extent to which Australian companies provide returns to shareholders in the form of franked dividends versus unfranked dividends versus capital gains. Indeed, Officer’s analysis implicitly assumes that all profits are perpetuities which are distributed as franked dividends.

It is also worth noting that Officer provides little guidance on whether γ is an economy wide parameter, or whether it is a company specific parameter which can vary between companies. In the derivation of cost of capital formulas, γ appears to be a company specific parameter. In his discussion of estimating the cost of capital, and economy wide parameter appears to be implicit in the CAPM formulation of his equation (18).

While is it is possible to rewrite the vernacular in the fashion proposed by Officer, it must be asked whether utilising a terminology which is at variance with everyday expression and with tax law jargon aids understanding or confuses. It must also be asked whether, since γ is unknown (although presumably obtainable in principle from tax records or estimated in other ways), this is the best approach to defining the cost of capital as Officer implicitly argues. Finally, as will be shown subsequently, there is no

3 A more general approach, albeit unnecessary as argued later in this paper, would be to define γ as the reduction in personal tax payments following from the payment of $1 of tax by companies. This would allow for dividend policies which do not involve a 100 per cent payout of franking credits.

4 Note that this does not mean that γ would be zero for unfranked dividends, if they were to be considered. γ is a concept relating to the valuation per dollar of franking credits received by an investor.

5 Officer (page 4) denotes X_E as dividends and franking credits as T(X_o -X_D), and defines profit going to shareholders as X_E = X_E' +γT(X_o-X_D), which implies a 100 per cent payout rate of profits in the form of franked dividends.
theory supporting Officer’s assertion that his approach is the correct (or a superior) one.

**Cash Flow Relationships and the Cost of Equity Capital**

Officer’s analysis is conducted for the special case in which free cash flows of the company have a constant expected value in perpetuity. Not only does this approach simplify calculations by enabling use of the perpetuity valuation formula, it also enables the analyst to use net operating income after taxes for an unlevered firm and free cash flow available to suppliers of capital interchangeably.

Officer divides company cash flows (operating income) \( X_o \) into the three shares of government \( X_g \), debtholders \( X_d \), and equity holders \( X_e \), shown in equation 1 as

\[
X_o = X_g + X_d + X_e \quad \text{Equation 1}
\]

where the government share is given as

\[
X_g = T(X_o - X_d)(1-\gamma) \quad \text{Equation 2}
\]

and the equity share consequently given as

\[
X_e = (X_o - X_d)(1-T(1-\gamma)). \quad \text{Equation 3}
\]

This includes both the cash component of dividends received \( (X_o - X_d)(1-T) \) plus the utilised value of imputation credits distributed \( (X_o - X_d)T\gamma \). Consequently, when Officer defines the cost of equity capital after company tax \( r_E \) from the identity

\[
S = X_o / r_E \quad \text{Equation 4}
\]

the figure is a “partially-grossed up” cost of equity capital. If, for example, \( \gamma=1 \) the return measure is fully grossed up by the value of franking credits distributed. If \( \gamma=0.5 \), only half of the franking credits distributed are included in calculating the rate of return.

---

6 Copeland and Weston (1988, Chapter 13 Section A, part 1) provide a good outline of the relevant relationships.
While there is nothing logically inconsistent with this approach, it needs to be emphasised that Officer’s $r_E$ is not equivalent to the traditional definition of the cost of equity capital after company tax, where company tax is the entire company tax bill paid. The relationship between the traditional definition (denoted here by $\rho_E$) and Officer’s measure is given by:

$$\rho_E = \frac{r_E (1-T)}{(1-T(1-\gamma))}$$  \hspace{1cm} \text{Equation 5}

This is important for several reasons. One is that it underpins his claim that “this does not imply that an imputation tax affects the cost of equity capital, which is measured on an after-company tax basis but before personal tax” (page 8). Since this involves a comparison of $\rho_E$ pre-imputation with $r_E$ post imputation (given Officer’s definition of company tax under imputation), it is consistent with $\rho_E$ having fallen following the introduction of imputation. A second reason for focusing on this relationship is that it is helpful in understanding some of Officer’s derived relationships between cash flows and the cost of capital to which we now turn.

The traditional WACC approach

Officer derives the WACC corresponding to the “standard after-tax definition of cash flows” given by $X_o (1-T)$ in his equation (7) as:

$$r_i = r_E \frac{S}{V} \frac{(1-T)}{(1-T(1-\gamma))} + r_D \frac{D}{V} (1-T)$$  \hspace{1cm} \text{Equation 6}

This will look unfamiliar to those used to undertaking capital budgeting using that cash flow definition. The reason is simply the one noted earlier: Officer’s definition of the cost of equity after company tax is one where equity returns are “partially-grossed up”. If the traditional measure of returns after company tax ($\rho_E$) is used, then substitution of equation 5 into equation 6 leads to:

$$r_E = \frac{\rho_E (1-T+T\gamma)}{(1-T)} = \rho_E + \rho_E T\gamma = \rho_E + \tau E \text{ as per Officer’s equation (17).}$$

For example, if pre imputation $\rho_E = 0.20$, and post imputation $r_E = 0.20$, with $T = 0.36$ and $\gamma=0.5$, then the implied post imputation value of $\rho_E = 0.20 (0.64)0.82 = 0.156.$
\[ r_i = \rho_E \frac{S}{V} + r_D \frac{D}{V} (1 - T) \]  
Equation 7

which is the familiar definition of the WACC.

*There is thus no need to alter the traditional approach to capital budgeting, although the appropriate value to use for the cost of equity capital for projects which involve Australian company tax payments and thus distributable franking credits can be expected to have changed (fallen) following the introduction of imputation.*

This is not to say that the introduction of imputation has not posed major problems for financial analysts. First, it is necessary to make some estimate of the appropriate cost of equity capital to use, knowing that historical information drawn from the pre (and perhaps even post) imputation environment cannot be used without modification. Second, the cost for capital for projects which are otherwise similar, but involve different amounts of Australian company tax payments (as could occur if one were an overseas project and one a domestic project) will differ.

**Officer’s Preferred Measure**

Officer argues for a new definition of the cost of capital which “replaces the effective company tax rate \( T \) with \( T(1-\gamma) \)” (page 1), and appears to recommend the use of his approach (ii) where cash flows to be valued are measured as \( X_0(1-T(1-\gamma)) \). As he demonstrates this involves use of a WACC measure given in his equation (10) as

\[ r_{ii} = r_E \frac{S}{V} + r_D \frac{D}{V} (1 - T(1 - \gamma)) \]  
Equation 8

In principle (as discussed later), there is no problem associated with utilising this approach to determine whether to accept or reject a project, but it must be asked whether it is a preferred approach. Implementing this approach requires (i) adjustment of the cash flows to incorporate the average value of franking credits \( \gamma \) (ii) adjustment of the cost of debt involving \( \gamma \), and (iii) estimation of the “partially-grossed up” cost of equity \( r_E = \rho_E(1-T)/(1-T(1-\gamma)) \). Adjustments (i) and (ii) involve an explicit adjustment using the unknown (but perhaps estimable) \( \gamma \) of the company in
question. The third item on this list is particularly problematic and deserves special attention.

Officer suggests using the CAPM (expressed in terms of returns measured in this partially grossed up fashion) to estimate \( r_E \) (or \( r' \) in the notation he uses in that part of the paper). This CAPM equation (Officer’s equation 18) takes the form:

\[
E(r_{jt}) = r_n + [E(r_{mt} + \tau_{mt}) - r_n] \beta_j
\]

This has the advantage that, under the reasonable assumption that imputation does not affect required returns after all tax (or after company tax -as that is defined by Officer), a market risk premium (the term in square brackets on the RHS of Equation 9) equivalent to the historical pre imputation risk premium can be used. This appears to involve some economy wide \( \gamma \) measure whose relationship to \( \gamma \)'s of individual assets is unclear. A more fundamental problem is that the individual equity returns on the LHS of equation (9) involve (possibly) different company specific values of \( \gamma \).

There is, to my knowledge, no asset pricing equation which has been derived from theory which would lead to such a formulation, nor explain why different companies have different \( \gamma \) values.

In contrast to Officer’s approach, the traditional approach can be implemented in a straightforward manner, and can be based on well specified CAPM equation built up from an optimizing model derived from returns after all tax. (See, for example, Monkhouse (1993), Brailsford and Davis (1995a,b).) The difficulty with that approach is that historical estimates of the market risk premium derived from a classical tax system are unlikely to be of much use in the imputation environment.

If, as Officer claims, all approaches are equivalent provided consistent measures of cash flows and cost of capital are used, it must be asked why one would be preferred over any other. The answer must presumably be that all measures are not equivalent in all respects, as we demonstrate subsequently.

**The Relevance of \( \gamma \)**
Officer argues that “\( \gamma \) is the value of personal tax credits” and in other work (Hathaway and Officer, 1992) has attempted to estimate \( \gamma \) by comparing dividend drop-off rates for shares paying franked and unfranked dividends. It must be asked what role \( \gamma \) plays in asset pricing and cash flow valuation. It \( (\gamma) \) is a company or economy wide average which is applicable to virtually no investor. For foreign investors\(^9\) and tax exempt domestic investors, \( \gamma = 0 \), while for tax paying domestic investors \( \gamma = 1 \), unless poor tax planning leaves them with zero tax payable but unused franking credits. Since asset pricing is driven by the decisions of marginal investors, it is unclear what role \( \gamma \) plays in determining the cost of equity for any individual firm.

One response to this criticism might be that, at a practical level, knowing \( \gamma \) enables the analyst to obtain better estimates of the cost of equity capital for Australian companies under imputation. As is well known, deriving a company’s cost of equity capital using a model such as the CAPM requires an estimate of the market risk premium (the excess of the expected return on the market portfolio of risky assets over the risk free interest rate), and such estimates are typically based on historical values. The introduction of imputation in 1987 is likely to have reduced that premium, when measured using traditional cost of capital measures, by some unknown amount.

Officer’s approach appears to get around this problem, although at the expense of introducing the new unknown value \( \gamma \). Measuring equity returns using the “partially grossed up” approach favoured by Officer gives an expression (Officer’s equation 15) for the return on equity of:

\[
 r_t = (p_t - p_{t-1} + d_t + \gamma \cdot C_t) / p_{t-1}
\]

which comprises capital gains \( (p_t - p_{t-1}) \), dividends (both franked and unfranked, \( d_t \)), and the value \( (\gamma) \) of franking credits \( (C_t) \) distributed with franked dividends. This

\(^9\) The argument here assumes that foreign investors are unable to gain any value from receipt of franked dividends. In practice, they may be able to sell franking credits on some “gray/black” market, and there may be various features of international tax treaties which mean that franking credits received reduce...
“partially grossed up” measure of return under the imputation system will be subject to tax at the investor’s personal tax rate. It is thus directly comparable with the traditional measure of return \( r = (p_t - p_{t-1} + d_t) / p_{t-1} \) prevailing prior to the introduction of imputation, in that both are subject to the investor’s personal tax rate in deriving the after-all tax rate of return. If the after-all tax rate of return required by investors on the market portfolio (in excess of the after tax risk free return) has not changed with the introduction of imputation, historical estimates (ie before imputation was introduced) of the market risk premium (based on the traditional measure) will provide a suitable estimate of Officer’s “partially grossed up” market risk premium. Any analyst wishing to continue using traditional measures of returns will need to lower their estimate of the market risk premium by the valuation of tax credits paid (expressed as a percentage of the value of the market portfolio). In Officer’s notation, the market risk premium measured traditionally will have fallen by \( \tau_t = \gamma C_t / p_{t-1} \).

This would seem to suggest that even those analysts favouring the traditional approach cannot escape the curse of Officer’s \( \gamma \). However, that is not so. Only if the analyst wishes to estimate the market risk premium by making some adjustment to the pre-imputation historical estimate does \( \gamma \) become relevant. Alternatively, the analyst may prefer to adopt an \textit{ex ante} estimate of the market risk premium or, given that we have now eight years of post imputation observations, use the historical market risk premium as measured using post 1987 data.

None of these alternatives is perfect, but neither is the approach advocated by Officer for several reasons. One problem is that although Officer’s approach avoids the need for a new estimate of the market risk premium, it introduces other difficulties. In particular, in applying the CAPM, it is necessary to “partially gross up” the equity return of the company under consideration. This involves the use of Officer’s \( \gamma \), which in his CAPM specification is an \textit{estimate} of some market wide average use of franking credits. The assumption that the shareholders of any specific company have the same their tax bill in their home country. Those possibilities do not alter the fact that the relevance of an economy wide average \( \gamma \) for asset pricing and valuation is questionable.
utilisation rate of franking credits is an extremely brave one, not founded on any published empirical evidence.

More general problems arise from the implicit assumption that other factors driving the market risk premium have not changed. It is well known that dividend payout ratios have increased following imputation, so that the composition of returns between dividends (franked and unfranked) and capital gains has changed. Since the tax treatment of these components of return is not equal, and significant changes to the tax treatment of capital gains occurred in 1985, the extrapolation of historical returns is fraught with danger.

Perhaps the most fundamental problem is the one referred to earlier. The $\gamma$ for any investor must, with some minor exceptions take the value of either 0 or 1. This implies that the marginal (price setting) investors in companies will have $\gamma$'s of one or zero. The average $\gamma$ provides no information to individual Australian companies about the returns required by the marginal, price setting, investors of relevance to them, although the average $\gamma$ may in some sense describe how much the equity premium (measured traditionally) for the Australian market will have changed vis a vis overseas following the introduction of imputation. It is not clear, however, that there is any well defined theory behind this story, and in any event it does not imply a need to use Officer’s suggested approach to capital budgeting rather than the traditional approach. Indeed, as we now argue, Officer’s apparently preferred approach to capital budgeting is not superior to the traditional approach, while several other alternative cost of capital formulations derived by him and suggested as suitable are subject to serious concerns regarding the accuracy of the net present value estimates generated.

The irrelevance of “consistency” proofs

A fundamental aspect of Officer’s approach (also relevant to Officer, 1981) is that his demonstrations of consistency requirements are all based on an initial starting point of a zero NPV. He commences with the market value for an asset, defines a set of cash
flows associated with that asset at some point in the tax payment cycle, and derives the cost of capital measure which equates the present value of the cash flows with the market value. This means that each identified approach should, if applied consistently, correctly value an asset whose NPV is zero, and will lead to similar accept or reject decisions. But it does not mean that each approach will give the same estimate of present value for a project with cash flows which has a non-zero net present value. While it can be shown that each approach can be structured so as to give the same, correct, value, some of those approaches are non-operational - in that they require prior knowledge of the project NPV.

This is easily demonstrated as follows, using for simplicity the case of a classical tax system. Consider a project with cash flows in perpetuity of $C$, subject to company tax at rate $T$. The project involves an initial outlay of $I$, and is small relative to the overall size of the company, so that the impact of a non zero NPV on the capital structure of the company (reflected in the debt/market value ($D/V$) or equity/market value ($S/V$)) can be ignored. When the unlevered cash flows $C(1-T)$ are discounted at a WACC of $r_o = r_E(S/V) + r_D(1-T)(D/V)$, as in the traditional analysis, the project has a non-zero NPV of $P$.

$$\frac{C(1-T)}{r_o} - I = P$$

Equation 11

10 This was demonstrated by Nantell and Carlson (1975) who also point out that while the same accept-reject decisions will be made using different measures of the cost of capital in conjunction with consistently defined cash flows, the capital structure which minimises the cost of capital will differ depending upon the measure used. Thus, even though the choice might be irrelevant for capital budgeting, it could lead to choice of a non-optimal capital structure, since minimum cost of capital according to some measures will not be coincident with maximum firm value.

11 This assumption is made simply to avoid complicating the issue with concerns about whether capital structure weights used in calculating the WACC for a positive NPV project should incorporate the project value using replacement or reproduction value. See Copeland and Weston (1988, Chapter 13, Section A, Part 3).
Rearranging equation 11, using the fact that interest cash flows (X_D) for the project are given by r_D I(D/V) it is simple to show that

$$\frac{C(1 - T) + X_D T}{r_E \frac{S}{V} + r_D \frac{D}{V}} = I + P(1 - \frac{r_D \frac{D}{V} T}{r_E \frac{S}{V} + r_D \frac{D}{V}})$$

Equation 12

As equation 12 demonstrates, the alternative valuation approach which allows for the interest tax shield in the cash flows rather than in the cost of capital leads to a different (incorrect) value for the NPV, unless the NPV (given by P) is zero (or trivially the company is all equity financed). This is also demonstrated in Appendix A using a simple example.

The obvious criticism of this demonstration of non-equivalence is that the debt tax shield used in the numerator of equation 12 is based on the replacement cost of the investment (I). If the project has a non-zero NPV and the company’s market value leverage is to be maintained constant, the extra debt capital used by the company (and the interest tax shield) should be calculated using the increase in market value of the company (ie the reproduction value of the investment). Appendix B provides a proof of the equivalence of the two approaches once appropriate allowance is made for this, but a fundamental assumption required in demonstrating this equivalence needs to be noted. *If the “correct” interest tax shield is to be allowed for in the cash flows being evaluated, it is necessary to know a priori the NPV of the project. The alternative approach is thus impractical for calculating the NPV of a project.*

Thus, while alternative cost of capital approaches can be shown to be equivalent in principle, they are not necessarily equivalent in practice. If replacement values are

$$12 \frac{C}{r_o} = I + P$$

Multiplying through by $r_o = r_E \frac{S}{V} + r_D (D/V)(1-T)$

$$C(1 - T) = (I + P)(r_E \frac{S}{V} + r_D \frac{D}{V}(1 - T))$$

$$C(1 - T) + r_D I \frac{D}{V} T = (I + P)(r_E \frac{S}{V} + r_D \frac{D}{V}) - P r_D \frac{D}{V} T$$

Noting that $X_D = r_D I(D/V)$ and dividing both sides by $(r_E(S/V)+r_D(D/V))$ gives equation 11.
used in calculating interest tax shields, the alternative approaches will give an incorrect NPV estimate. If reproduction values are to be used in calculating interest tax shields, the alternative approaches will give the same value, but the “true” NPV estimate must be known a priori - making the approach which allows for interest tax shields in the cash flows non-operational.

Two consequences follow from this.

First, it is desirable to determine which approach can be implemented in practice to provide the correct estimate of NPV values, and necessary if mutually exclusive projects are to be considered\textsuperscript{14}. In the context of Officer (1994), his approaches (iii) and (iv) are non-operational for the purposes of calculating NPV, since the cash flow definitions involve interest tax shields. This leaves his approaches (i) and (ii) which appear to be equivalent. It is known that the traditional approach (i) generates “correct” NPV results even when a “Miller equilibrium” (which is consistent with the dividend imputation system) is assumed or when cash flows do not take the form of a perpetuity (provided that capital structure is maintained over time at its optimal level)\textsuperscript{15}. Officer has not demonstrated that his suggested approach (ii) is superior in any way for the case of perpetual cash flows, where approaches (i) and (ii) are equivalent, and the applicability of Officer’s alternative (ii) in other circumstances has not been demonstrated.

\textsuperscript{13} Note that the proof in the text and the example in the appendix assume that the financing mix of the project used in calculating its WACC is the same as that for the company. This is common, and is consistent with the notion that there exists an optimal capital structure.

\textsuperscript{14} If mutually exclusive projects are being considered, it is necessary to be able to compare NPV values.

\textsuperscript{15} See, for example Taggart (1991), where in panel A of exhibit 3, the traditional WACC is derived as the consistent approach, in the case where corporate tax and personal tax - with differential tax rates for debt and equity income- is considered. Since imputation can be interpreted as a situation in which the personal tax rate on equity income (t_e) differs from that on interest (t_p) (by defining t_e as \((t_p-t_c)/(1-t_c)\) to reflect the value of tax credits), this suggests that the traditional approach remains appropriate under imputation. Note however that considerable complexities are introduced once projects are of finite life with varying cash flow streams and uncertain tax payments.
Second, it is apparent from Officer’s equations that any numerical value for $\gamma$ can be fed into them and maintain consistency. If we are only concerned about accept-reject decisions, any value of $\gamma$ can be used, in any of Officer’s formulae. If we are concerned with accuracy of NPV estimates, any value of $\gamma$ can be used in Officer’s alternatives (i) and (ii) where cash flows are perpetuities. The only relevance of $\gamma$ then is if knowing it provides greater accuracy of actual NPV estimates for projects with cash flows which are not perpetuities. Since we do not know that Officer’s suggested approach provides correct valuations in these cases even if a “correct” $\gamma$ is used, it is extremely premature to advocate the use of the “partial gross up” approach when the true value of $\gamma$ is not known.

The use of $\gamma$ in Officer’s suggested approach is further complicated by the need to then adjust cash flows for a project being considered by a company using the “$\gamma$-factor”. As noted earlier, it is by no means apparent that it is appropriate to use this historically based adjustment factor. If marginal investors in this company happen to value imputation credits at $\gamma$ per dollar, the approach might give the right answer. But that cannot be guaranteed.

**Conclusion**

Officer’s suggestion that a new cost of capital measure be used in valuation methodology does not appear to have any firm grounding in finance theory. While it is possible to interpret part of the tax paid by a company as a prepayment of investor-level tax and part as “company tax”, it is not clear that an approach to capital budgeting based on that perspective is preferable to alternative approaches nor warrants the potential confusion resulting from the unconventional definition of company tax involved. More seriously, Officer’s approach has not been shown to provide correct estimates of present value, and if used in valuation methodology could lead analysts into error.
Appendix A

This appendix provides a simple demonstration of the fact that although alternative definitions of cash flow and cost of capital can be utilised in deciding to accept or reject a proposal, they are not otherwise equivalent if replacement values are used, and provide no grounds for advocating one approach over another as a method of evaluating cash flow streams with non-zero present value.

Consider a project which promises a perpetual income stream of \( X = \$100 \) p.a. The company tax rate \( T_c \) is 30\%, the cost of equity (traditionally measured) is \( r_e = 12\% \), the cost of debt is \( r_d = 8\% \) and the company is 50\% equity financed \( (k = E/(D+E) = 0.5) \). Interest expense is denoted by \( X_d \) and equals \( r_d(1-k)I \) where \( I \) is the initial outlay on the project. The project is small relative to the size of the company, so that the impact of a positive NPV project on the aggregate debt-equity ratio of the company can be ignored). Two alternative ways to value the NPV of that project will be considered. First, the cash flows of the project, calculated as if it were unlevered, ie \( X(1-T_c) \), can be discounted at the weighted average cost of capital given by \( k_r e + (1-k)r_d(1-T_c) \) . (This is Officer’s case (i) for a classical tax system). Second, the cash flows after tax (including tax deductions due to interest ), ie \( X-T_c(X-X_d) \) can be discounted at the weighted average cost of capital given by \( k_r e + (1-k)r_d \). (This is Officer’s case (iii) for a classical tax system). The Table below provides values for the NPV of the project for different assumed values for the outlay (I) associated with the project. Thus, for example, if the initial outlay is $400, the NPV using approach (i) is $395.5 and using approach (iii) is $348. Note, however, that both approaches give an NPV of zero when the initial cost is $795.5, and both would give the same accept-reject decision.

<table>
<thead>
<tr>
<th>Initial outlay (I)</th>
<th>Interest Cash Flow ( X_d = r_d(1-k)I )</th>
<th>Present Value (case (i))</th>
<th>Present Value (case (iii))</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>16</td>
<td>395.5</td>
<td>348</td>
</tr>
<tr>
<td>600</td>
<td>24</td>
<td>195.5</td>
<td>172</td>
</tr>
<tr>
<td>795.5</td>
<td>31.82</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>800</td>
<td>32</td>
<td>-4.5</td>
<td>-4</td>
</tr>
<tr>
<td>1000</td>
<td>40</td>
<td>-204.5</td>
<td>-180</td>
</tr>
<tr>
<td>1200</td>
<td>48</td>
<td>-404.5</td>
<td>-356</td>
</tr>
</tbody>
</table>
Appendix B

This appendix demonstrates that the alternative definition of project NPV based on after tax cash flows (including interest tax shields) is correct when reproduction values are used for determining project financing. However, it also demonstrates the need to know a priori the net present value of the project, thus making the approach impractical.

A classical tax system is assumed, and the two NPV formulas are given below, where $C$ is the project cash flow before tax, $T$ is the company tax rate, $I$ is the initial outlay, $k$ is the (traditional) WACC, $r_D$ is the cost of debt, $(D/V)$ is the target leverage, and $k^*$ is the alternative measure of WACC as given in equation 11 of the text. Thus:

$$k^* = r_E(S/V) + r_D(D/V)$$

so that $k^* = k + r_D T(D/V)$.

The traditional NPV formula is given by

$$NPV = \frac{C(1-T)}{k} - I$$

and the alternative formula is given by

$$NPV^* = \frac{C(1-T) + Tr_D(I+NPV)}{k^*} - I$$

However, this assumes that the firm increases its debt only by the amount $I(D/V)$, which would lead to a decline in the market value leverage of the firm. To maintain a constant leverage ratio, the firm must increase debt by an amount equal to $(I+NPV)(D/V)$

Making this substitution in the formula for $NPV^*$ we have:

$$NPV^* = \frac{C(1-T) + Tr_D(I+NPV)}{k^*} - I$$

by using the definition of NPV. Multiplying by $k^*$ we have
\[ k'NPV^* = C(1 - T)[1 + rT \frac{D}{V} / k] - k'I \]
\[ [k + rT \frac{D}{V}]NPV^* = C(1 - T)[1 + rT \frac{D}{V} / k] - (k + rT \frac{D}{V})l \]
\[ k[1 + rT \frac{D}{V} / k]NPV^* = C(1 - T)[1 + rT \frac{D}{V} / k] - k(1 + rT \frac{D}{V} / k)l \]
\[ kNPV^* = C(1 - T) - kl \]
\[ NPV^* = \frac{C(1 - T)}{k} - l = NPV \]

While this demonstrates the equality of the two formulae once reproduction values are used for determining the interest tax shield, it must be noted that the \( {NPV}^* \) formula requires pre-existing knowledge of the project NPV. It can thus not be used independently to obtain the correct NPV estimate.
References


Brailsford T J and K T Davis (1995b) “Understanding Imputation” JASSA, December, 14-17


