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Corporate Capital Management and Protection of Warrant-holder Claims

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ABSTRACT

Corporate capital management techniques such as rights issues, share buy-backs, and returns of capital can adversely affect the value of company issued warrants. Commonly prescribed adjustments to warrant terms (both by exchanges on which such warrants trade and in issue documentation) do not generally protect warrant holders against loss of value from such capital management activities. This paper develops an analytical framework which is used to derive the appropriate adjustments to the terms of company-issued warrants for a range of corporate capital management activities.

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Introduction

This paper uses a novel unifying framework to derive the correct value preserving adjustments required for company issued warrant terms following various corporate capital management actions. It also demonstrates that inappropriate adjustments are commonplace, implying significant value transfers from warrant to shareholders.

Returns of capital to shareholders and share buy-backs have become increasingly common components of corporate capital management programs in recent years in many countries. The reasons are varied and include both shareholder tax benefits and consequences for stock market valuation arising from signaling and information effects. Similarly, rights issues and private placements (at a discount to the current stock price) have, for many years, been a popular feature of capital management programs as a method for raising equity capital.

Such actions affect the total market valuation of the company both directly, when they occur through the cash inflow or outflow, and also indirectly when announced, through information and signaling effects. They also have valuation implications for other stakeholders in the firm because of the change in company assets occasioned by the outflow or inflow of cash. The value of debt claims may be altered because of the change in leverage of the company, and creditors will generally have attempted to protect themselves against adverse effects by covenants and provisions in debt agreements.

Holders of company issued options (warrants) and hybrid securities such as convertible notes may also find the value of their claims affected both through cash flow and dilution effects. These stakeholders may be protected by provisions written into the issue documentation for the securities, or by rules of the Stock Exchange on which they are traded. Such provisions typically require adjustments to be made to contract terms and conditions when certain company initiated actions (such as returns of capital or rights issues) occur.

In this paper we derive the correct adjustments required to company-issued warrant terms to ensure protection of value, and demonstrate that many commonly used adjustments are incorrect. (In contrast, the correct adjustment formula is typically

specified for the case of exchange traded options and third party issued warrants). Apart from the important policy conclusions which emerge from the analysis, a contribution of the paper is the development of a unifying framework which enables the adjustments to be derived and compared for different types of capital management activities.

Since the paper focuses on valuation changes for company issued warrants, Section 1 of the paper briefly reviews the commonly accepted valuation models for such securities and illustrates the magnitude of changes in warrant value which can arise from corporate capital management activities. In Section 2, we apply some basic principles of financial engineering to develop a framework within which the effects of a range of capital management activities can be assessed. The derivation of the correct adjustments to warrant terms is provided in Section 3. Section 4 provides information on the adjustments prescribed in a sample of warrant documentation and stock exchange rules to demonstrate a widespread lack of consistency with the correct adjustments derived in the previous section. Section 5 concludes with implications for policy and suggestions for future research.

1. Warrant Valuation

Warrants are (typically call) options issued by companies which provide the holder with the right to purchase stock in the company at some specified exercise price. Because they are issued by the company, and involve the creation of additional shares if exercised, they differ from exchange traded, third party issued options, because of the dilution effect which occurs on exercise.

Techniques for valuing company issued options have been available in the finance literature for over thirty years, following the pioneering work of Black and Scholes (1973) and Merton (1974). While there have been significant debates over some aspects of the precise formulas (and parameters therein) to be applied to the valuation of warrants, Handley (2002) demonstrates how these apparently conflicting approaches can be reconciled.

For simplicity, we focus on the valuation of warrants for an unlevered firm as found in Stoll and Whaley (1993) or Hull (2005).[1] Table 1 presents the relevant notation, where each warrant gives the right to purchase one share.

Table 1: Warrant Pricing Notation	
Symbol	Interpretation
S (s)	Aggregate (individual) market value of shares on issue
W (w)	Aggregate (individual) market value of warrants on issue
V (= S+W)	Aggregate market value of the (unlevered) firm
n_s	Number of shares on issue
n_w	Number of shares issued if warrants are exercised
$\gamma = n_w /(n_s + n_w)$	Dilution factor due to presence of warrants
K	Aggregate amount paid by warrant holders to exercise their option
X	Strike price per warrant
T	Expiry date of warrants
σ	Volatility (standard deviation) of return of total firm value
$\alpha = n_w/n_s$	Ratio of number of warrants to number of shares on issue
c	$ return of capital
x	$ discount to share market price of rights issue

Stoll and Whaley (1993, p 332-334) demonstrate that the aggregate warrant value can be written as:

$$W = \gamma V N(d_1) - (1-\gamma) K e^{-rT} N(d_2) \qquad (1)$$

where,

$$d_1 = \frac{ln\left(\frac{\gamma V}{(1-\gamma)K}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \qquad (2)$$

$$d_2 = d_1 - \sigma\sqrt{T} \qquad (3)$$

and N(.) is the cumulative normal distribution function. Note that because the value of the firm (V = S+W) enters the right hand side of equation (1), the equation cannot be solved explicitly for W and requires the use of numerical techniques.

Alternatively, the problem can be formulated on a per share and warrant basis. The value of the warrant giving the right to purchase one new share can be derived (see Hull, 2005) as

[1] Application to the case of a levered firm (with risk-free debt) is straightforward, but would complicate the algebra without providing additional insight.

$$w = \frac{1}{1+\alpha}\left[(s+\alpha w)N(d_1) - e^{-rT} X\, N(d_2)\right] \quad (4)$$

where,

$$d_1 = \frac{ln\left(\frac{s+\alpha w}{X}\right)+(r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \quad (5)$$

d_2 is derived from d_1 as in equation (3) and $s+\alpha w$ equals total firm assets per outstanding share.

To illustrate the effect of corporate capital management activities on warrant value, consider what happens if the company makes a return of capital to shareholders of \$c per share or \$C = n_sc in aggregate. The total market value of the company falls from \$V to \$(V-C). Hence, if no change in the warrant contract terms is made, the aggregate value of warrants on issue will fall to a new value W*, which is calculated by replacing V with (V-C) in equations (1) – (3) above. Similarly the value of total firm assets per outstanding share fall to \$(s+αw-c), which is used in calculating the new individual warrant price w* from equations (4) and (5).

Table 2 provides some illustrative values of the change in warrant value for a range of parameter values. For example, column (1) demonstrates that for a stock price of \$10.00, a strike price per warrant of \$10 and a warrant per share ratio of $\alpha = 0.2$, the value of the warrant to purchase one new share is w = \$0.978.[2] Around 53 percent of the warrant value is lost (w* = \$0.458) if the firm undertakes a return of \$1 in capital to shareholders and does not adjust the warrant contract terms. Column (2) illustrates that the percentage loss is not particularly sensitive to the initial warrant per share ratio. Column (3) indicates a higher percentage loss in the case of a higher strike price, while column (4) indicates that the percentage loss declines with increased volatility. Column (5) shows that the loss is quite sensitive to the size of the capital return.

Summarizing, it can be seen that the percentage decline in warrant value is larger for cases where: the dilution effect (α) is larger; the exercise cost relative to firm value (K/V) is larger; the return of capital (c) is larger. The greater the asset volatility the

[2] This compares with a straight call value, for the same parameters of \$1.045, which is higher because of the absence of any dilution effect on conversion.

more the warrant is worth, but the smaller the percentage reduction in value of the warrant when the firm makes a return of capital. The effects are not insignificant and for reasonable parameter values imply substantial losses to warrant holders.

Table 2: Illustrative warrant values.

This table shows the loss in warrant value when the company makes a return of capital and there is no adjustment to the warrant value. The risk-free interest rate is assumed to be 5 percent and the time to maturity of the warrant is one year.

	1	2	3	4	5
	Base case	**Larger dilution (rel to 1)**	**Higher strike (rel to 1)**	**Higher volatility (rel to 3)**	**Larger return of capital**
stock price (s)	10	10	10	10	10
strike price (X)	10	10	12	12	10
volatility of assets (σ)	0.2	0.2	0.2	0.3	0.2
Return of capital (c)	1	1	1	1	2
Warrants per share ($\alpha=n_w/n_s$)	0.2	0.4	0.2	0.2	0.3
Dilution factor (γ)	0.167	0.286	0.167	0.167	0.231
Firm value/exercise amount (V/K)	5.098	2.592	4.190	4.218	3.428
Old warrant value (w)	0.978	0.923	0.284	0.616	0.949
New warrant value (w*)	0.458	0.417	0.099	0.321	0.151
Percentage of warrant value lost	53.2%	54.8%	65.2%	47.9%	84.1%

The fact that the change in warrant value does not appear to be highly sensitive to the dilution factor (as shown by comparison of columns (1) and (2) of table 2), suggests that use of the adjustment formula commonly used for exchange traded (third party) options (ETO's), where dilution is zero, might be a suitable method of protecting warrant holders. This adjustment involves an increase the conversion ratio from one share per warrant to s/(s-c) shares per warrant and also a reduction in the strike price to X(s -c)/s. Brown (1989) pointed out that such an adjustment was required for rights and bonus issues, and that the "traditional" adjustment procedure specified at that time by several exchanges, involving a reduction in the option strike price by the value of the rights issue, was not correct.[3]

Figure 1 illustrates across a range of values for α (the ratio of number of warrants to number of shares) the accuracy of the ETO based approximation suggested above. (The values chosen for other parameters correspond to those of column (1) of Table 2: s=\$10; X=\$10; σ = 0.2; T=1; r = 0.05; c=\$1). The figure illustrates that applying the

[3] He also noted that "similar procedures should be adopted in company-issued options, convertible debt and in other similar contracts" but did not specify the precise nature of the required procedures.

ETO adjustment formula to company issued warrants results in a gain to warrant holders! This arises because the ETO adjustment assumes a fall in the share price of $c. When company issued warrants exist and no adjustment of warrant contract terms is made, the share price fall is less than $c, because of the transfer of value from warrant holders to stock holders. Basing the required adjustment of contract terms on an assumed share price decline of $c thus leads to an overestimate of the adjustment required. However, if there are very few warrants on issue, such that the dilution effect is small, the ETO adjustment may not be unreasonable.

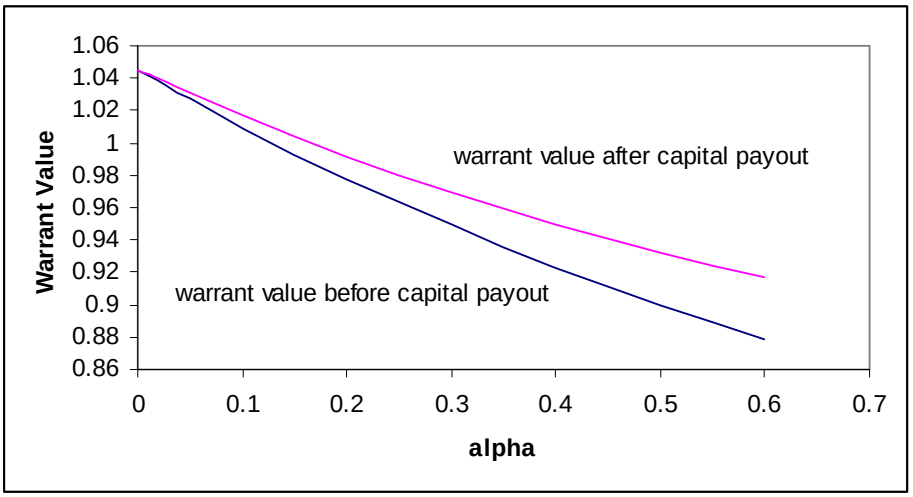

Figure 1: Change in warrant value after return of capital to shareholders and (incorrect) use of ETO adjustment formula

2. *Corporate Capital Management: A Framework for Analysis*

Common techniques of corporate capital management include rights issues (often at a discount to market price)[4], returns of capital to shareholders, and share buy-backs.[5] The objective of this section is to illustrate how the effect of these apparently diverse actions on warrant prices can be analyzed and compared within a simple framework.

In comparing the potential effects on warrant holders, the salient features of these actions outlined in Table 3 are as follows. Rights issues involve an injection of cash

[4] Since we are not concerned in this paper with actions which redistribute corporate value between different shareholders, we are able to treat private placements as equivalent to rights issues.

[5] For simplicity, we focus here primarily on self tender (off-market) share buy-backs at a premium to the market price. We ignore complications raised by tax issues which may lead to buy-backs occurring at a discount to market price, as occurs in Australia (Brown and Davis, 2006), and which involve tax induced changes in the share price.

(which may be zero in the case of a bonus issue) into the company by equity holders accompanied by an increase in the number of shares on issue. A share buyback involves a return of cash to equity holders and a reduction in the number of shares on issue. A return (injection) of capital involves an outflow (inflow) of cash to equity holders (with no change in shares on issue).[6]

Table 3. Share and Cash Flow Effects of Corporate Capital Management Actions

Action	Cash Flow	Number of Shares
Bonus Issue (Share Split)	0	+
Rights Issue (Private Placement)	+	+
Buyback	-	-
Return of Capital	-	0
Injection of Capital	+	0

To unify the subsequent analysis and highlight relevant differences, we derive correspondences between rights issues, buy-backs and returns of capital which enable us to compare their effects on warrant values. In doing so, our aim is to show that these actions can be decomposed into (at most) two components which each involve either a change in the number of shares on issue only, or a change in the total assets of the company only. Since (as we demonstrate in the next section) the effect of each of these components on warrant values can be derived analytically, this will enable analytical solutions for the appropriate adjustments in the more complex cases of capital management.

We first derive the relationship between the effects on warrant values of rights issues and share buybacks. To do this, consider the effect of the following two hypothetical strategies.

- Strategy 1 is a rights issue of Δn_s shares at a discount of \$x per share to the current market price (of \$s) by a company with n_s shares on issue. This leads to an increase in shares on issue (of Δn_s) and a net injection of cash into the company (of $\Delta n_s(s-x)$).
- Strategy 2 is a buy-back of Δn_s shares at a discount of \$x per share to the current market price (of \$s) by a company with n_s shares on issue. This leads

[6] Injections of capital are unlikely in publicly traded firms (due to problems of reaching shareholder agreement) but are in principle possible (which is all that is required for our analysis) and do occur in private companies.

to a decrease in shares on issue (of Δn_s) and a net outflow of cash from the company (of $\Delta n_s(s-x)$).

It is immediately apparent that these strategies are identical but opposite. Hence, letting ΔW_z represent the change in the value of company issued warrants arising from strategy z:

$\Delta W_{Strategy1} = - \Delta W_{Strategy2}$

$\Delta W_{\text{rights issue at \$x discount}} = - \Delta W_{\text{buyback at \$x discount}}$

and we have demonstrated Proposition 1.

> *Proposition 1: A rights issue and a share buyback for the same number of shares and at equal discounts to the market share price have equal but opposite effects on the value of outstanding company warrants*

The implication of Proposition 1 is that the adjustment required to warrant terms to protect warrant holders will take the same form (although of opposite direction) for both rights issues and buybacks undertaken at a discount to the market price.

While buybacks at a discount to the market price are uncommon (and typically occur for tax based reasons), proposition 1 provides the basis for deriving the effect on warrant values of a buyback at a premium to the market price. Consider the case of a buyback of Δn_s shares at a premium of \$x per share to the current market price (of \$s) by a company with n_s shares on issue. This leads to a decrease in shares on issue (of Δn_s) and a net outflow of cash from the company (of $\Delta n_s(s+x)$). Let strategy 3 be defined as:

- Strategy 3 is a return of capital of $2\Delta n_s x/(n_s - \Delta n_s)$ per share to $(n_s - \Delta n_s)$ shareholders.

It is clear that the buyback at a premium of \$x is equivalent to the combination of Strategy 2 followed by Strategy 3. Both lead to a reduction in the number of shares of Δn_s and an outflow of cash of $\Delta n_s(s+x)$. Hence,

$\Delta W_{\text{buyback at \$x premium}} = \Delta W_{\text{buyback at \$x discount}} + \Delta W_{Strategy\ 3}$

This leads to proposition 2.

- *Proposition 2:* The effect on warrant values of a buyback at a premium (of $x) is equal to the combined effect of a buyback at a discount (of $x) and a return of capital (of $2\Delta n_s x/(n_s - \Delta n_s)$ per share to $(n_s - \Delta n_s)$ shareholders).

The import of Proposition 2 is as follows. The correct adjustment to warrant terms for a return of capital can be easily derived (as will be shown shortly). If the correct adjustment to warrant terms for a rights issue at a discount is known, Proposition 1 enables derivation of the required adjustment for a buyback at a discount. Combining these two adjustments gives the correct adjustment for a buyback at a premium.

We now consider the relationship between rights issues and returns of capital. To do so, we compare two hypothetical strategies, strategy 1 (as previously defined) and strategy 4 defined by:

Strategy 4 is the joint transaction of a rights issue of Δn_s shares at no discount to the market price accompanied by some return of capital of $c per share to all $(n_s+\Delta n_s)$ shareholders. This leads to the same increase in shares on issue (of Δn_s) as for Strategy 1 and a net injection of cash into the company of $\Delta n_s s - c(n_s + \Delta n_s)$.[7]

The net injection of cash, and thus increase in company value, will be the same in both cases if:

$$\Delta n_s (s-x) = \Delta n_s s - c(n_s + \Delta n_s) \quad (6)$$

or

$$c = x. \Delta n_s /(\Delta n_s + n_s). \quad (7)$$

Note that the right hand side of equation (7) is the textbook theoretical value of a right. Thus, if the return of capital (c) is calibrated according to equation (7), Strategies 1 and 4 are ultimately identical in terms of corporate value and number of shares on issue. This enables derivation of the relationship between adjustments for rights issues and capital returns, which highlights an important difference between the case of ETOs and company issued warrants.

[7] In the case where there are no other stakeholders in the company (and thus no potential transfer of value between stakeholders), the order of the return of capital and zero discount rights issue could be reversed. Because the share price would drop by $c following the return of capital, the net injection of funds would be $- cn_s + (s-c) \Delta n_s = s \Delta n_s - c(n_s+\Delta n_s)$. In the case where other (warrant) stakeholders exist, the equivalence is not immediate, since the share price change following the return of capital may differ from $c if warrant values are affected.

. Before proceeding to that derivation it is useful to illustrate that difference between adjustments required for ETO and company issued options. Consider first the simple case where the company has no warrants on issue, but where there are third party options on issue. Since both strategy 1 and strategy 4 are identical if $c = x.\ \Delta n_s /(\Delta n_s + n_s)$, they will both have the same effect on the value of the third party options. Note also that the value of such options does not depend directly upon the number of shares on issue, and will be unaffected by zero NPV capital management activities. Since an issue of new shares at the current market price is a zero NPV transaction, the value of third party issued options would not be affected by such an action. Consequently, letting ΔO_z represent the change in the value of third party options arising from strategy z:

$$\Delta O_{Strategy1} = \Delta O_{Strategy4}$$

$$\Delta O_{\text{rights issue at \$x discount}} = \Delta O_{\text{rights issue at \$0 discount}} + \Delta O_{\text{return of capital of \$x}\Delta ns /(\Delta ns + ns)}$$

$$= \Delta O_{\text{return of capital of \$x}\Delta ns /(\Delta ns + ns)}$$

Since the change in the third party issued option value for a rights issue at a discount of $x is the same as that for a return of capital of $c = x\Delta n_s$ /($\Delta n_s + n_s$), the adjustment applied to the terms of a third party issued warrant for a rights issue of $x should be the same as that applied for a return of capital of $c = x\Delta n_s$ /($\Delta n_s + n_s$). The latter term is the theoretical value of the right, hence we have established Proposition 3.

> *Proposition 3: The change in the value of a third party issued option will (unless contract terms are adjusted) be the same for a return of capital and for a rights issue at a discount if the return of capital is of the same size as the theoretical value of the right.*

This result finds reflection in the approach adopted by many exchanges for adjusting the terms of ETOs in response to rights issues and returns of capital. For a return of capital of $c per share, the conversion ratio is increased from one share per warrant to s/(s-c) shares per warrant and the strike price reduced to X(s -c)/s. In the case of rights issues at a discount to market price, the same adjustments are made with the theoretical value of the right used instead of the return of capital.

Consider now the case where n_w company issued warrants are outstanding with an aggregate strike price of $K = n_wX. Both Strategy 1 and Strategy 4 will have the same effect on the value of the warrants, such that:

$$\Delta W_{Strategy1} = \Delta W_{Strategy4}$$

$$\Delta W_{\text{rights issue at \$x}} = \Delta W_{\text{rights issue at \$0}} + \Delta W_{\text{return of capital of \$x}\Delta ns /(\Delta ns + ns)}$$

where ΔW_z is the change in warrant value from strategy z. Now, however, the impact on warrant value of a rights issue at a zero discount is no longer zero, but is negative, because of a dilution effect. Consequently,

$$\Delta W_{\text{rights issue at \$x}} < \Delta W_{\text{return of capital of \$x}\Delta ns /(\Delta ns + ns)}$$

and the decline in the company issued warrant value caused by a rights issue at a discount of $x is greater than the decline caused by a return of capital of amount $c = x. Δn_s /($\Delta n_s + n_s$). This demonstrates Proposition 4.

> *Proposition 4: The adjustments required to company issued warrant terms for a rights issue and a return of capital of equal value to the theoretical rights value, are not equivalent as is the case for third party issued warrants.*

We now consider the effect of a rights issue at a discount, by utilizing hypothetical strategy 5, defined by:

Strategy 5 is a bonus issue to shareholders of Δn_s shares accompanied by a separate capital injection of $y per share. This leads to a net injection of cash of $y(n_s + \Delta n_s)$ and increase in shares on issue of Δn_s.

Consider strategies 1 and 5. Strategy 1 is the rights issue of Δn_s shares at a discount of $x or $\Delta n_s(s-x)$ in total. Strategy 5 is the combination of a bonus issue of Δn_s shares followed by a capital injection of $y per share or $(n_s+\Delta n_s)y$ in total. Provided that $\Delta n_s(s-x) = (n_s+\Delta n_s)y$, ie that the capital injection $y = \Delta n_s(s-x)/(n_s+\Delta n_s)$, both have identical effects on corporate value and number of shares on issue.

Consequently:

$$\Delta W_{Strategy1} = \Delta W_{Strategy5}$$

$$\Delta W_{\text{rights issue at \$x}} = \Delta W_{\text{bonus issue}} + \Delta W_{\text{injection of capital of \$(s-x)}\Delta ns /(\Delta ns + ns)}$$

Note that the injection of capital is the converse of a return of capital. Hence we are able to state Proposition 5.

Proposition 5: The effect on warrant value of a rights issue at a discount can be decomposed into an effect reflecting a change in the number of shares (as represented by a bonus issue) and an effect reflecting the cash injection component (which is the converse of the effect resulting from a return of capital).

We now use these results to derive the required adjustments to warrant terms to protect holders against various capital management actions.

3. The Correct Adjustment Formulae

Our objective in this section is to use the results of the previous section to determine the appropriate adjustment to company issued warrant contract terms to prevent the effect which would otherwise occur on the value of warrants on issue when a company makes a return of capital to its shareholders, a rights issue, or a share buyback.

In the case of ETOs, the adjustment which needs to be made to contract terms to protect option holders from capital management activities is well known and easily derived. It is well known (Merton, 1973) that the Black-Scholes option pricing formula is linear homogeneous in the stock price S and option strike price X.[8]. Consequently if a capital return of \$c dollars per share were made, the correct value-preserving adjustment to the option terms can be easily derived by noting that:

$$\frac{S}{S-c} * BS(S * \frac{S-c}{S}, X * \frac{S-c}{S}, r, \sigma, T) = \frac{S}{S-c} * BS(S-c, X * \frac{S-c}{S}, r, \sigma, T) \quad (8)$$
$$= BS(S, X, r, \sigma, T)$$

where BS(.) represents the Black-Scholes pricing formula.

Examining equation (8) it can be seen that in this case of exchange traded options (where there is no dilution effect from option exercise) the appropriate adjustment is to increase the conversion ratio from one share per option to S/(S-c) shares per option and also reduce the strike price to X*(S-c)/S. At the post-capital return share price of S-c, the adjusted option contract will have the same value as before the return of capital. Propositions 1 and 3 from the previous section provide the basis for the application of similar style adjustments in the cases of rights issues and share buy-backs.

[8] If both S and X are multiplied by a scalar (z) the value of the option also changes by the factor z.

In the case of company issued warrants, despite the implicit function nature of equations (1) and (4), the equations remain linear homogeneous in the stock price, warrant price and the strike price. Using equation (4),

$$w = \frac{1}{1+\alpha}\left[(s+\alpha w)N(d_1) - e^{-rT} X\, N(d_2)\right], \qquad (4)$$

consider the implications of multiplying both sides by the scalar z. Since[9]

$$z\frac{1}{1+\alpha}\left[(s+\alpha w)N(d_1) - e^{-rT} X\, N(d_2)\right] = \frac{1}{1+\alpha}\left[z(s+\alpha w)N(d_1) - e^{-rT} zX\, N(d_2)\right]$$

$$zw = \frac{1}{1+\alpha}\left[z(s+\alpha w)N(d_1) - e^{-rT} zX\, N(d_2)\right]$$

Substituting $\left(\frac{s+\alpha w - c}{s+\alpha w}\right)$ for z (where w and s are the pre-capital return warrant and share values respectively) and rearranging gives

$$w = \left[\frac{s+\alpha w}{s+\alpha w - c}\right]\frac{1}{1+\alpha}\left[(s+\alpha w - c)N(d_1) - e^{-rT} X\left(\frac{s+\alpha w - c}{s+\alpha w}\right)N(d_2)\right], \qquad (9)$$

where,

$$d_1 = \frac{\ln\left(\frac{s+\alpha w}{X}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T} \qquad (10)$$

Thus there is a simple value preserving adjustment given by equation (9) where the appropriate adjustment is to increase the conversion ratio from one share per warrant to s+αw/(s+αw-c) shares per warrant and also reduce the strike price to X((s+αw-c)/(s+αw)). We have thus derived Proposition 6.

Proposition 6: The adjustment required to protect company issued warrant holders against changes in value due to a return of capital to shareholders of $c per share is to increase the conversion ratio from one share per warrant to (s+αw)/(s+αw-c) shares per warrant and reduce the strike price to X((s+αw-c)/(s+αw)).

To derive the correct adjustment formula for rights issues at a discount we draw on Proposition 5 which demonstrated the equivalence with a bonus issue plus a capital

[9] Note that d_1 depends upon the ratio (s+αw)/X which is unaffected by multiplying both numerator and denominator by z.

injection. Since a capital injection is the converse of a return of capital (and the adjustment can thus be derived using Proposition 6, it is necessary to derive the appropriate adjustment for a bonus issue. In this case, it is more convenient to use the aggregate warrant valuation equation (1)

$$W = \gamma V N(d_1) - (1-\gamma) K e^{-rT} N(d_2) \tag{1}$$

where

$$d_1 = \frac{ln\left(\frac{\gamma V}{(1-\gamma)K}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \tag{2}$$

and $\gamma = n_w/(n_s+n_w)$. We first note that if the bonus issue involves an increase in shares on issue from n_s to $n'_s = n_s + \Delta n_s$ then d_1 will be unchanged if K and n_w are changed to K' and n'_w such that:

$$K' = \frac{n'_w n_s}{n_w n'_s} K \tag{11}$$

Making such a change, the new aggregate warrant value will be

$$\begin{aligned} W' &= \gamma' V N(d_1) - (1-\gamma') K' e^{-rT} N(d_2) \\ W' &= \frac{\gamma'}{\gamma}[\gamma V N(d_1) - (1-\gamma) K e^{-rT} N(d_2)] = \frac{\gamma'}{\gamma} W \end{aligned} \tag{12}$$

For $W' = W, \gamma'/\gamma = 1,$ or $n'_w / n_w = n'_s / n_s$, so that the warrant holders should receive an proportional increase in the number of warrants equal to that received by shareholders. Note that while this implies that the aggregate strike amount K is unchanged, it requires a reduction in the individual warrant strike price from

$X = K/n_w$ to $X' = K'/n'_w = K/n'_w = Xn_w/n'_w$. Consequently we have derived proposition 7.

Proposition 7: In the case of a bonus issue the adjustment required for protection of warrant holders is to increase the number of warrants proportionally to the increase in shares (by the factor $(1 + \Delta n_s/n_s)$*, and decrease the strike price by dividing it by the same factor, such that the aggregate strike amount is constant.*

The intuition behind this result is straightforward. The value of the company has not changed, and the warrant holders should have the right to purchase the same proportion of the company for the same total outlay. Hence the number of warrants needs to be increased and the strike price reduced to keep the total exercise amount constant.

We are now in a position to derive the appropriate adjustment for a rights issue at a discount, which we have previously demonstrated to be equivalent to a bonus issue together with an injection of capital (i.e. a negative return of capital). Specifically, for a rights issue of Δn_s shares at a discount of \$x per share, the capital injection accompanying the bonus issue in the replicating transactions was previously shown to be equal to:

$$y = \Delta n_s(s-x) / (n_s+\Delta n_s). \tag{13}$$

Noting that the number of warrants and strike price have already been adjusted to n'_w and X' (and that the aggregate share and warrant value will have been unaffected) the second set of adjustments required are derived using the return of capital/ capital injection adjustment. Noting that α (the ratio of warrants to shares) has been kept constant in the adjustment for the bonus issue component, we have:

$$\begin{aligned} n''_w &= n'_w\left(\frac{s+\alpha w}{s+\alpha w+y}\right) = n'_w\left(\frac{s+\alpha w}{s+\alpha w+\frac{\Delta n_s(s-x)}{n_s+\Delta n_s}}\right) = n_w\frac{n_s+\Delta n_s}{n_s}\left(\frac{s+\alpha w}{s+\alpha w+\frac{\Delta n_s(s-x)}{n_s+\Delta n_s}}\right) \\ X'' &= X'\left(\frac{s+\alpha w+y}{s+\alpha w}\right) = X'\left(\frac{s+\alpha w+\frac{\Delta n_s(s-x)}{n_s+\Delta n_s}}{s+\alpha w}\right) = X\frac{n_s}{n_s+\Delta n_s}\left(\frac{s+\alpha w+\frac{\Delta n_s(s-x)}{n_s+\Delta n_s}}{s+\alpha w}\right) \end{aligned} \tag{14}$$

This leads to Proposition 8.

Proposition 8: In the case of a rights issue to shareholders of Δn_S shares, where n_s shares were previously on issue, at a discount of \$x per share to the market price, the required adjustment to warrant terms involves two steps. First the warrant conversion ratio should be increased proportionately to the share issue and the strike price decreased similarly. Second, the warrant conversion ratio should be then reduced by a factor reflecting the injection of funds from the rights issue, and the strike price increased similarly.

Proposition 8 is illustrated for the case of a one for 5 rights issue ($\Delta n_s/n_s = 0.2$) at a discount of x = \$2 to the current market price of s = \$10, where there are α = 0.2

warrants per share on issue each with a strike price of $10. (The values chosen for other parameters correspond to those of column (1) of Table 2: $\sigma = 0.2$; T=1; r = 0.05, and the initial warrant value is w = $0.978.) In stage one of the adjustment, the conversion ratio is increased from 1 to 1.2 and the strike price reduced to $10/1.2 = $8.33. In stage 2, the conversion ratio is reduced from $n_w = 1$ to:

$$n_w'' = 1.2\left(\frac{10+0.2(0.978)}{10+0.2(0.978)+\frac{0.2(10-2)}{1+0.2}}\right) = 1.2\left(\frac{10.1956}{11.76423}\right) = 1.039993$$

$$X'' = 8.33\left(\frac{10+0.2(0.978)+\frac{0.2(10-2)}{1+0.2}}{10+0.2(0.978)}\right) = 8.33\left(\frac{11.52893}{10.1956}\right) = 9.419359$$

The validity of this adjustment is confirmed by calculating the new aggregate warrant value using the initial values given in Table 2, where the stock price was $10 and the warrant value was $0.978, with 2 warrants outstanding per share with a strike price of $10.

4. Adjustments in Practice: Stock Exchange Rules and Warrant Agreements

In this section we provide evidence that the standard adjustments made to warrant terms following capital management activities are not consistent with the correct adjustments derived in the previous section. Consequently, there is substantial value transferred between stakeholders in the company.

We focus[10] at this stage on the Australian market.

The ASX Listing Rules (ASX, 2005) address the issue of adjusting company issued warrant terms for a return of capital in paragraph 7.22.3 which states:

"*In a Return of Capital* – the number of options must remain the same, and the exercise price of each option must be reduced by the same amount as the amount returned to each ordinary security".

An example is provided in that paragraph in which a return of capital of $1 per share implies that an existing option to purchase one share should have its strike price reduced from $2 to $1. Note that the adjustment required does not depend upon any of the parameters entering the warrant valuation formula, such as the relative size of the

[10] The data collection for this section is currently being undertaken, involving examples of warrant adjustment terms in a number of major international markets.

required exercise amount to the total firm value or the dilution factor. The adjustment is inconsistent with the correct adjustment derived in Proposition 6.

The prescribed adjustment for a rights issue is given in paragraph 6.22.2 of the Listing Rules shown in Appendix 1. Utilising the notation adopted in this paper and assuming that each option entitles the holder to one share and that the share is not ex-dividend, the adjustment proposed is that the strike price X *may* be reduced by an amount of

x. $\Delta n_s / (n_s + \Delta n_s)$.

Alternatively, the rules specify that if the terms of the option propose an alternative adjustment, that adjustment may be used.

If can be seen that the approach specified by the ASX assumes that the effect on a company issued warrant's value is the same for both a rights issue and for a return of capital of an amount equal to the theoretical value of the right. As shown that is not correct. Moreover, as also shown the appropriate adjustments are not the simplistic one of adjusting only the strike price by the suggested amount.

A commonly used alternative is for the warrant prospectus to provide for early exercise by the warrant holder in the event of a capital management event such that, by conversion, the investor will participate in the benefits otherwise accruing to shareholders (such as the discount on a rights issue). It is obvious that early exercise involves loss of the time value implicit in an warrant's value, and that this adjustment thus does not provide full protection for the warrant holder.

5. Conclusions

In this paper, we have demonstrated that the adjustments to company issued derivative securities specified in warrant documentation and by stock exchanges to protect investors when company initiated capital events occur are often incorrect. We have shown that the typical investor protection clause found in issue documents for such securities, allowing early exercise if certain capital events occur, and permitted for use by many exchanges, does not provide full protection. We have illustrated that investors in company issued warrants can lose significant value by application of these procedures.

Consequently one conclusion of this paper takes the form of a recommendation for changes to stock exchange rules such that theoretically correct adjustment of terms of

company issued derivatives occur in future. As shown in Section 3, unlike in the case of third party issued derivatives, the adjustment for rights issues and for return of capital are not equivalent.[11]

There is scope for further research which extends our analysis in several ways. First, there are many other types of complicated derivative securities on issue, and the theoretical adjustments which should, and actual adjustments which have, been applied to the terms of such securities deserve study. Second, empirical work examining the actual extent of value transfers between shareholders and warrant holders from cases of capital management is warranted. Third, exposure to expropriation from capital management activities due to incomplete protection introduces a risk which should be reflected in traded warrant prices. Fourth, many companies undertaking capital management strategies have non-listed company issued executive options outstanding. Whether value destroying adjustments to the terms of such options have also occurred is an interesting avenue for future research.

[11] Moreover, unless the correct adjustments are made to protect holders of such securities, the theoretical value of rights used in adjusting terms of third party issued derivatives is incorrect for companies with company issued derivatives on issue, giving rise to a (small) bias in the (otherwise correct) adjustment specified.

APPENDIX 1
ASX Listing Rules, Chapter 6, pp 6090-6010 (24 October 2005)
http://www.asx.com.au/ListingRules/chapters/Chapter 06.pdf

6.22 An option may confer the right to a change in its exercise price, or a change to the number of [+]underlying securities over which it can be exercised, in any of the following cases. (Introduced 1/7/96. Origin: Listing Rule 3G(1)(c)(iii). Amended 1/7/97.)

6.22.1 Introduced 1/7/96. Origin: Appendix 29. Deleted 1/7/97. Refer rule 6.22.2A.

6.22.2 If there is a [+]pro rata issue (except a [+]bonus issue) to the holders of the [+]underlying securities, the exercise price of an option may be reduced according to the following formula.

$$O' = O - \frac{E[P - (S + D)]}{N + 1}$$

O' = the new exercise price of the option.

O = the Old exercise price of the option.

E = the number of underlying securities into which one option is Exercisable.

Note: E is one unless the number has changed because of a bonus issue.

P = the average market Price per security (weighted by reference to volume) of the underlying securities during the 5 trading days ending on the day before the ex rights date or ex entitlements date.

S = the Subscription price for a security under the pro rata issue.

D = the Dividend (in the case of a trust, Distribution) due but not yet paid on the existing underlying securities (except those to be issued under the pro rata issue).

N = the Number of securities with rights or entitlements that must be held to receive a right to one new security.

Introduced 1/7/96. Origin: Appendix 29. Amended 1/7/97, 1/7/98.

6.22.2A As an alternative to using the formula in rule 6.22.2, if the option was issued with the approval of holders of ordinary securities, and is not in a class of quoted options, the exercise price or number of underlying securities may change if there is a pro rata issue (except a bonus issue) to the holders of the underlying securities in accordance with the formula contained in the terms of the option. The notice of meeting must have contained a report by an independent expert on the effect of the proposed changed, and a voting exclusion statement.

Introduced 1/7/96. Origin: Listing Rule 6.22.1.

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