THE SYSTEMATIC RISK OF DEBT: AUSTRALIAN EVIDENCE*

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ABSTRACT

This paper examines systematic risk (betas) of Australian government debt securities for the period 1979 - 2004 and makes three contributions to academic research and practical debate. First, the empirical work provides direct evidence on the systematic risk of government debt, and provides a benchmark for estimating the systematic risk of corporate debt which is relevant for cost of capital estimation and for optimal portfolio selection by asset managers such as superannuation funds. Second, analysis of reasons for non-zero (and time varying) betas for fixed income securities aids understanding of the primary sources of systematic risk. Third, the results cast light on the appropriate choice of maturity of risk free interest rate for use in the Capital Asset Pricing Model and have implications for the current applicability of historical estimates of the market risk premium. Debt betas are found to be, on average, significantly positive and (as expected) closely related, cross sectionally, to duration. They are, however, subject to significant time series variation, and over the past few years the pre-existing positive correlation between bond and stock returns appears to have vanished.

Keywords: Systematic Risk, Government Debt, Bond Returns

I. INTRODUCTION

The importance of estimating systematic (market) risk (betas) for equity securities in order to estimate the cost of capital or for performance measurement purposes is well known, and such estimates are readily available from professional providers or easily calculated. Less readily available, and less well understood, are systematic risk estimates (debt betas) for fixed interest securities such as government and corporate bonds. This is particularly the case in Australia where development of a deep and liquid government (and even more so corporate) bond market is a relatively recent phenomenon. Elsewhere there have been numerous studies (such as Elton, Gruber and Blake, 1995) which have examined corporate and government bond returns and their sensitivities to multiple risk factors.

Estimating debt betas is important for at least four reasons. First, there is considerable practical debate over the extent to which corporate debt exhibits non-zero systematic risk but, to date, little evidence which is relevant for assessing that debate. This is an important consideration in estimation of a company's weighted average cost of capital for two reasons. One is that the cost of debt finance will depend upon its risk characteristics. The second is that common procedures used to estimate the cost of equity from information about "comparable" companies involves adjustments for differences in financial leverage which need to allow for the systematic risk of debt.[1] These issues have been a source of contention in access pricing for regulated industries (ACCC, 2003). By providing estimates of government debt betas, and analyzing their relevance for estimating corporate debt betas, this paper contributes to more informed debate on the issue.

Second, as Cornell (1999) has noted, analysis of the reasons for the existence of non-zero betas for debt securities which have non-varying cash flows may aid understanding of the primary sources of systematic risk in general. Fixed interest rate government securities have no default risk and no uncertainty about future cash flows. Consequently, all systematic risk must be due to either economy wide changes in required (expected) rates of return, or covariance between changes in expectations about cash flows on risky assets and required rates of returns, rather than the result of changes in expected cash flows of the asset in question (bonds). Examining systematic risk of government bonds avoids the need to adopt decomposition methods such as those used by Campbell and Mei (1993) or Vuolteenaho (2002) in their studies of equity betas, while providing some evidence on the relevance of systematic changes in required rates of return.

Third, investigation of government bond betas can also shed light on the appropriate choice of maturity for the risk free interest rate (r_F) to be used in applications of the Capital Asset Pricing Model (CAPM) and in estimation of the Market Risk Premium (MRP). This has also been a contentious issue in applying finance theory based on an unspecified horizon period to practical problems (in areas such as cost of capital estimation, valuation and capital budgeting) involving multi-period cash flows.

Fourth, superannuation fund managers make asset allocation decisions involving, *inter alia*, choices between debt and equity. Optimal portfolio allocation requires knowledge of the systematic risk of various asset classes considered for inclusion in the portfolio.

The remainder of the paper is structured as follows. Section II briefly reviews the CAPM as background for the subsequent discussion. Section III outlines the reasons for non zero debt betas and makes conjectures about characteristics of debt betas. Section IV discusses the data used in the study, paying particular attention to the methods used to develop appropriate estimates of holding period returns on government bonds. Section V presents and discusses the results from estimating betas for government bonds. Section VI provides some concluding comments and suggestions for further research.

II. THE CAPM AND MRP

The CAPM provides a theory of the pricing of risky assets, and thus of the determinants of expected returns on those assets. Within that framework, denoting expectations by E(.) the expected return on risky asset *k* is given by:

$$E(r_k) = r_F + \beta_k MRP \qquad (1)$$

where

$$\beta_k = cov(r_M, r_k) / var(r_M) \qquad (2)$$

$$MRP = E(r_M) - r_F \qquad (3)$$

and r_F is the risk free interest rate and r_M is the return on the market portfolio of risky assets. For the holding period (horizon) assumed in this framework, the risk free asset should have either a non-stochastic holding period rate of return or alternatively, in the zero beta CAPM of Black (1972), could be an asset with a return uncorrelated with that on the market portfolio. A finding of a non-zero beta for government bonds with maturity greater than the holding period assumed for risky asset returns in applications of the CAPM or in calculation of the MRP, would render choice of that maturity asset as the risk free security inappropriate. As Booth (1999) notes, if long term bonds have a non zero beta, the equity premium measured relative to the return on long term bonds (r_D) is given by:

$$E(r_e) - E(r_D) = MRP(\beta_e - \beta_D) \qquad (4)$$

where β_e and β_D are the systematic risks of equity and debt respectively, measured against a broad market portfolio of risky assets (inclusive of debt), and MRP is the risk premium on that broad portfolio.[2]

Use of the yield to maturity on such an asset (a ten year bond, for example) as the risk free rate (as is often advocated in practice) would be particularly inappropriate. Not only is the asset not a zero beta asset, its yield to maturity measures return over a different (longer) holding period). As Jarrow (1978) demonstrates, yield to maturity is not a good measure for a bond's expected rate of return over a shorter horizon.

In practice, the return on the risky market portfolio is typically proxied by the return on the stock market. In principle, this portfolio should include all risky assets including bonds whose market price can fluctuate over the assumed holding period. In this paper, an augmented market portfolio incorporating Commonwealth Government Bonds is constructed, and results compared to those obtained using the more conventional market portfolio proxy. Some implications of the observed changes in debt betas and the share of equity in the augmented market portfolio over time for the equity premium are outlined.

III. DETERMINANTS OF DEBT BETAS

The debt beta of a fixed income security reflects the covariance between the holding period return on that security and on the market portfolio of risky assets (as defined explicitly in equation 2 above). Even though fixed income, default free, securities (such as government bonds) have certain cash flows if held to maturity, actual returns over any shorter holding period are stochastic. For example, if interest rates rise, prices of fixed income securities fall and the return over the holding period spanning that event (calculated as the percentage change in price plus any interest cash flows received) will be lower than the yield to maturity prevailing at the start of the period.

Thus fixed income securities can (and should) be treated, like equities, as a risky asset in the CAPM model, if their maturity[3] differs from the horizon assumed in this single period model. However, there is little guidance from theory as to what is the length of the period involved in the CAPM. Consequently, the appropriate maturity for the risk free rate used in the CAPM and in calculation of the MRP is open to debate. In practical applications, risky asset returns are often calculated assuming a holding period (horizon) of one month (or less).

Consistency with the structure of the CAPM implies choosing as a measure of the risk free rate the yield to maturity on a debt security of equivalent maturity to the holding period assumed for risky assets. However, many researchers and practitioners often ignore this requirement and use yield to maturity on a debt security with a much longer term to maturity. If the holding period return on such a security has an expected value equal to the yield to maturity and, second, is uncorrelated with the return on the market portfolio, this may not create problems since the zero beta CAPM of Black (1972) can be applied. However, the first of these conditions requires that the expectations hypothesis of the term structure of interest rates applies (contrary to available empirical evidence), while the second is also inapplicable (as will be subsequently demonstrated).

Regardless of the approach taken to choosing a risk free rate, an important empirical question is how debt betas vary with the maturity and issuer characteristics of those debt instruments.

Considering first the relevance of issuer characteristics to debt betas, the principal characteristic of relevance is that of credit risk. The promised cash flows of corporate bonds are subject to default risk and for this reason the yields to maturity (which are calculated assuming no default) of corporate bonds exceed those of similar maturity government bonds. Some part of that credit spread is compensation for expected loss due to non zero default probability and some part is a risk premium to reflect any systematic risk in actual returns if, for example, default (or expectation thereof) is correlated with market returns.[4]

Variability in holding period returns on corporate bonds (and thus corporate bond betas) will reflect changes in required yield to maturity which in turn reflects changes in required returns on risk free (government) bonds and/or in required credit spreads. If credit spreads have no systematic element (are uncorrelated with market returns), the beta of a corporate bond would be approximately equal to that of an equivalent maturity (duration) government bond. If credit spreads have some (positive) systematic component, corporate bond betas could be expected to exceed those on government debt. Determining the magnitude of that differential is an important research topic, although not feasible for the Australian market at present given the paucity of historical data on corporate bond yields. It could, however, be conjectured that the main determinant of corporate bond betas will result from the systematic element in government bond returns, since credit spreads reflect concerns about possible default over the remaining maturity of the bond rather than imminent default, and could thus be expected to have relatively low correlation with current market returns.[5]

Returning to the case of default free securities, a straightforward prediction about the relationship between different bond betas emerges from considering the characteristics of bonds.

As Cornell (1999) notes, if the only changes in the yield curve are parallel shifts then the beta (for a specified holding period) of a bond with modified duration[6] D should be approximately k times that of a bond with modified duration D/k. This follows from noting that the beta of a bond with modified duration D is defined as

$$\beta_{D,m} = cov(r_D, r_m)/var(r_m) \tag{5}$$

where r_D is the holding period return on the bond and r_m is the return on the market. Then using the duration approximation for calculating holding period returns as:

$$r_D = -D\Delta r \tag{6}$$

where Δr is the change (in basis points) in the level of interest rates, the beta can be expressed as

$$\beta_{D,m} = -Dcov(\Delta r, r_m)/var(r_m) \tag{7}$$

This demonstrates that differences in duration are the principal factor leading to different betas, provided that yield curve movements are parallel. It would thus be expected that a proportional relationship would exist between beta and duration.[7]

In practice, the yield curve does not always behave in this fashion. In fact, over the period considered here, interest rate targeting by the Reserve Bank has led to lower interest rate volatility at short maturities. This would tend to increase the sensitivity of beta to duration because $\mathrm{cov}(\Delta r^L, r_m) > \mathrm{cov}(\Delta r^S, r_m)$ where r^L and r^S are long and short term interest rates respectively. However, at the same time duration is a convex function of maturity for bonds with equal coupons. Consequently substituting maturity for duration will tend to offset the effect of interest rate targeting when the relationship between beta and maturity is considered. Hence, in addition to estimating and analysing debt betas the empirical work also provides evidence on the following conjecture:

Conjecture 1: The relationship between beta and maturity of bonds will be approximately proportional.

It is well known that estimated betas are not necessarily stable over time. Hence it is important to look at time series variation in estimated betas and identify causes for such variation. Changes in the variance of the return on the market, the denominator of equation 7, will influence estimated betas, but more interesting is the possibility of time series variation in $\text{cov}(\Delta r, r_m)$ in the numerator of the equation.

To examine $\text{cov}(\Delta r, r_m)$ it is useful to note that variation in the return on the market portfolio can essentially be decomposed into two sources – changes in expected future cash flows from the portfolio and changes in the discount rate (required rate of return) on that portfolio. Campbell (1991), for example, provides a linear approximation for unexpected stock returns involving the revision to expectations of future dividends (which is denoted here as u_d) and the revision to future required returns (u_h). In such a framework, -D. $\text{cov}(\Delta r, r_m)$ can be decomposed into two components as:

$$-D.cov(\Delta r, r_m) = -D[cov(\Delta r, u_d) - cov(\Delta r, u_h)] = Dcov[(\Delta r, u_h) - cov(\Delta r, u_d)] \quad (8)$$

The first of these components, $\text{cov}(\Delta r, u_h)$, would be expected to be positive, since it reflects covariance of the discount rates applied to shares and bonds. The second term, $\text{cov}(\Delta r, u_d)$, could be either negative or positive. If, for example, an increase in interest rates was generally correlated with an upward revision of expectations of future cash flows on the market portfolio, that covariance would be positive and could outweigh the first term. In that case, negative returns on bonds (due to the increase in interest rates) would be accompanied by positive stock returns (due to higher expected future cash flows) and the beta could be negative. In general, however, it might be expected that the positive correlation between discount rates applied to stocks and bonds would dominate, leading to positive bond betas. This leads to:

Conjecture 2: bond betas will typically be positive.

The effect of using a broader market index on the estimated values of bond betas is a priori uncertain, but likely to increase beta estimates because of two effects. First, inclusion of bond returns in the calculation of a market return would be expected to (marginally) increase the correlation between the returns on individual bonds and the market index and thus increase estimated betas. Second, including bond returns may reduce the variance of market returns (if bond returns are less variable than, and not perfectly correlated with, equity returns). With the decline in relative capitalisation of the bond market over the period, these effects should decline over time, leading to

Conjecture 3: Bond betas calculated using an augmented market index should be higher than those calculated using the All Ordinaries index, but the difference should be less in later years of the period studied.

IV. DATA[8]

Reliable estimation of historical debt betas, applicable to the current time, is constrained by two factors. One is the lack of availability of a long time series of reliable, market determined, yields on government debt. The secondary market in Australian government bonds developed significantly following financial deregulation in the early 1980s to the extent that the turnover rate per annum of the stock of securities on issue increased from below unity prior to 1983 to just under ten times by the start of the 1990s. Since then the turnover rate has increased markedly, and the Reserve Bank's policy of concentrating debt on issue into a small number of benchmark stock has meant that reliable market yields (albeit on a small number of available debt instruments) are available for use. Table I provides summary information on benchmark bonds. A second factor limiting the relevance and reliability of historical estimates is the change which occurred in the *modus operandi* of monetary policy at the start of the 1990s, when a policy of publicly announcing a target for the short term interest rate began. Pre-existing relationships between asset returns may not have been invariant to this change.

[Insert Table I]

To calculate bond betas it is necessary to specify the term to maturity and/or duration characteristics of the bond under analysis and to keep these characteristics constant over time.[9] Thus for example, calculation of the beta of a 10 year maturity bond requires a time series of monthly observations of the one-month holding period return on a bond with 10 years remaining to maturity at the start of the month and a specified coupon. However, at any point in time it is highly unlikely that there will be a bond trading with the desired characteristics and, given the fixed maturity date of bonds, the term to maturity (or duration) characteristics of any individual bond will change over time. As a result, it is necessary to calculate holding period returns on hypothetical bonds of specified maturities by interpolation using actual bonds whose maturities are near to and span that term to maturity. An interpolation process between bonds with different coupon rates, and where the "spanning" bonds change over time, will mean that the hypothetical bond of fixed maturity will not have a constant duration over time. However, any noise introduced by this process should be small since the induced duration changes for the hypothetical bond should not be too large or frequent.

This paper utilises two alternative methods to generate monthly holding period returns on government bonds. The first approach uses data for yields to maturity on actual bonds traded, which is available from the early 1990s onwards, and is referred to below as the *actual yield method*. Holding period returns are calculated directly for actual bonds on issue and interpolation then used to estimate the holding period return on hypothetical bonds of a given maturity (such as 10 years). While this should provide accurate estimates, a disadvantage is the relatively short time period for which data is available. The alternative approach uses the series constructed (by interpolation) by the Reserve Bank for yields to maturity on hypothetical bonds of specified (2, 5 and 10 year) maturities, and is referred to below as the *interpolated yield method*. This has the advantage of being available from the late 1970s, but has the potential disadvantage that process of interpolation of yields to maturity from bonds of varying maturity might create distortions when these are subsequently used to estimate monthly holding period returns.

By comparing results from these two alternative approaches, one minor contribution of this paper is to demonstrate that errors arising from the second approach are of second order of importance for empirical work of this sort (although not necessarily so for trading purposes). For the period July 1992 to December 2003 for which data was available for both approaches, the correlations between monthly holding period rates of return calculated using the alternate methods were all in excess of 0.98 for hypothetical 2, 5, and 10 year maturity bonds.[10] Consequently, only results from the second approach, which is outlined below are presented, and an outline of the first approach relegated to the appendix.

Bond Returns calculated using Interpolated Yield Data

This method of calculating monthly holding period returns commences with yield to maturity estimates for hypothetical bonds of specified maturities estimated by the Reserve Bank using linear interpolation. This series is available on a monthly basis from the late 1970s for bonds with 2, 3, 5, and 10 year maturities. The price of such hypothetical bonds was calculated each month using the specified yield to maturity and the Reserve Bank pricing formula for a bond with an assumed 8 per cent coupon rate of interest. The price of the same bond was recalculated one month later (assuming a month shorter remaining maturity) and the continuously compounded rate of return calculated. Thus if P^n_t and P^{n-1}_{t+1} are the price of an n month maturity bond at time t and the same (but now n-1 month maturity) bond at time t+1, the monthly holding period return is given by:

$$r^n_{t,t+1} = \ln (P^{n-1}_{t+1} / P^n_t)$$

Risk Free Interest Rate

The risk free rate of return for the one month holding period t to t+1 is defined to be the market determined yield observed at date t on a security of maturity 1 month. Two choices were available. Treasury Notes with 5 weeks to maturity were initially considered, but lack of data prior to November 1991 meant that 30 day Bank Bill rates were used instead. Returns were expressed as a continuously compounded return for the holding period h (in days), using the formula:

$$r_{t,t+h} = ln(1 + \frac{y_{t,t+h} h}{365})$$

where $y_{t,t+h}$ is the yield observed at t on a bill with h days maturity[11].

Market Portfolio Return

The return on the market portfolio was calculated in two ways. The first approach, typically used in empirical work, uses the All Ordinaries Accumulation Index published by the ASX, and available on a daily basis. The "h period" return on the market between date t and t+h, denoted by $r^m_{t,t+h}$ was calculated as a continuously compounded rate of return as:

$$r^m_{t,t+h} = ln(I_{t+h}/I_t)$$

where I_t is the accumulation index at date (end of month) t.

Since the market portfolio of risky assets should, in principle, include all risky assets (not just equities), a broader market index was constructed which also incorporates government bonds. (In the context of a one month horizon, these are risky assets). The index was constructed in the following way. First, the face value of Commonwealth Government bonds on issue was used to approximate the market capitalisation of bonds. Second, the holding period return on a bond with maturity equal to the average maturity of bonds on issue[12] was calculated by linear interpolation from holding period returns on 2 and 5 year, or 5 and 10 year bonds. The return on the broad market index was constructed as a capitalisation weighted average of the average bond return and the return on the All Ordinaries.

[Insert Figure 1]

Several features of this augmented index and its components warrant mention. First, as shown in Figure 1, at the start of the sample period (1979) bonds accounted for approximately 45 per cent of the broad index, declining to below 10 per cent at the end of the sample (2004). Thus, while incorporating bond returns into the market index may (and does) affect the bond betas calculated in earlier decades, the need for such an adjustment in recent years is significantly reduced. Second, the return on the broader index was highly correlated with the return on the All Ordinaries. The correlation coefficient over the entire period was 0.98 and above 0.96 for all 36 month sub periods. Third, the augmented index had a lower volatility (standard deviation of returns) of 14.9 per cent p.a. compared to 19.6 per cent p.a. for the All Ordinaries over this period.

V. RESULTS

This section presents the results from estimating debt betas using two alternative approaches. First, unconditional beta estimates are calculated for the case where beta is assumed constant over the entire sample period. Second, time varying betas are assumed and conditional beta estimates calculated using several alternative techniques.

Full Sample, Constant Beta, Estimates

Estimates of bond betas for maturities of 2, 5, and 10 years were calculated from OLS regression equations of the form[13]:

$$r_{it} - r_{ft} = \beta_{0i} + \beta_{1i}(r_{mt} - r_{ft}) + u_t \qquad (9)$$

for i = 2, 5 and 10, where $r_{it,}$ and r_{mt} are, respectively, the actual monthly holding period returns for period t to t+1 on the hypothetical i year bond and the market portfolio, and r_{ft} is the risk free rate for period t to t+1. Two alternatives, the All Ordinaries accumulation index and the augmented index were used for the market portfolio.

Beta estimates were calculated for the entire period 1979(12) – 2004(02), assuming constancy of beta over the entire sample. Table II presents the results.[14] The expectation of a positive relationship between bond betas and maturity is confirmed, and the relationship while not exactly linear is consistent with Conjecture 1 (that an approximately linear relationship would exist). Betas for all maturities are positive and significantly different from zero, consistent with conjecture 2. This suggests that, in the framework of Equation 8, positive correlation between required returns on debt and equity outweighs any positive covariance between interest rates and expected cash flows on equity.[15]

[Insert Table II]

The size of the bond betas is also of interest, with the beta of 10 year bonds being in the order of 0.15, that for 5 year bonds in the order of 0.10, and 0.046 for 2 year bonds when the All Ordinaries Index is used as the market portfolio. These are values with some economic significance. For 10 year bonds, if an equity premium of 5 per cent p.a. is assumed, a beta of 0.15 means that the expected one month holding period return would be 0.75 per cent p.a. higher than that on a one month security to compensate for the systematic risk. To place that estimate of the risk premium into an historical context, the difference between the geometric mean of observed monthly values for the yield to maturity for the 10 year bond of 9.45 per cent p.a. and that for the 30 day bank bill of 8.74 per cent p.a. was 0.69 per cent p.a.[16] If, on average, there was no expectation of higher or lower interest rates over the period, that difference of 0.69 per cent p.a. could be broadly interpreted as the observed risk premium corresponding to the CAPM expected premium based on the bond's beta.

The results also suggest that arbitrary separation of asset portfolio management into equity and bond portfolios is inappropriate since returns on both are affected by a common factor. Although the explanatory power of market returns to the determination of bond returns is generally less than 10 per cent (as indicated by the R squared values) the results do indicate that attempts to hedge bond portfolio returns which do not take account of this systematic risk may be unsatisfactory.

Consistent with conjecture 3, use of a broader market index results in larger bond beta estimates, and has better explanatory power as shown in Table II, panel B.

Time Varying Systematic Risk

The existence of time variation in asset betas is well documented as is the existence of a variety of techniques for modelling such time variation (see for example, McKenzie and Brooks, 1999, and Brooks, Faff and McKenzie, 2002). Three approaches were considered here for estimating conditional bond betas. They were (a) rolling regressions (b) a bivariate GARCH approach in the form of the BEKK model and (c) a Kalman Filter approach.

The rolling regression approach involved first estimating beta for a sample period (window) of 36 months from 1979:12 – 1982:11 to generate an historical beta estimate for 1982:11, then rolling the window forward by one month, re-estimating, and repeating the process, to generate a series of historical beta estimates. The problems with such an approach are twofold. First, there is a conceptual problem in the assumption that beta is constant over a 36 month window and possibly different for another 36 month window with which it overlaps. Second, the equal weight given to each observation means that shifts in beta will only be gradually reflected with a lag in the regression estimates.[17]

A variety of multivariate GARCH models have been proposed to estimate conditional variances and covariances of asset returns which can then be used to derive conditional beta estimates. The BEKK model (Engle and Kroner, 1995) is one such multivariate GARCH approach which imposes specific restrictions on the variance-covariance equations. A simplified restricted version involves assuming GARCH (1,1) processes of the form:

$$h_{ij}(t) = w_{ij}^2 + b_i b_j\, h_{ij}(t\text{-}1) + a_i a_j\, e_i(t\text{-}1)e_j(t\text{-}1) \text{ for } i, j = 1,2$$

where e_i is the residual for asset i, h_{ij} is the covariance between e_i and e_j, and w, b and a are parameters to be estimated. If asset j is the market portfolio, the condition beta estimate of asset i at time t can be derived as $h_{ij}(t) / h_{jj}(t)$.

This type of approach allows shocks (e_i, e_j) to have an immediate (next period) impact on variances and covariances and thus on the conditional beta estimates. However, it assumes constant autoregressive parameters, estimated over the whole sample period, for the evolution of variances and covariances. In contrast, the rolling regressions approach utilises only data from the current window.

A third approach considered was use of a Kalman filter state space model in which beta is assumed to be a time varying coefficient in Equation (9). A random walk process for beta was assumed.

The results of all three approaches were generally similar as shown in Figure 2 for the case of 10 year bond betas (calculated relative to the All Ordinaries Index), except for some divergences in the last few years which are discussed later. Hence only the rolling regression results are reported in detail.

[Insert Figure 2]

Figure 3 presents the time-varying beta estimates for 10 year bonds using both market indexes and for 2 and 5 year bonds using the equity market index. It is apparent the assumption of a constant beta over time is unfounded. The noticeable drop in betas in the late 1980s is only partly due to inclusion of the stock market crash of October 1987. Excluding the months around the crash from the calculations removes the sharp drop shown in figure 3, but betas still decline gradually until the early 1990s. Over the course of the 1990s, bond betas were consistently and significantly positive, with the 10 year bond beta (measured against the stock index) in the region of 0.2 – 0.4. The gap between bond betas measured using the two indices gradually declined, as conjectured (given the declining bond market capitalisation), to the point where there now appears little to be gained from using that augmented index.

[Insert Figure 3]

What is particularly striking about the results of the rolling regressions is the negative bond betas found for the latest years of the sample. Since early 2002, bond betas have been negative, although generally not significantly different from zero, until the last few subsamples. This can be seen for the case of 10 year bonds in Figure 4 which shows the rolling regression R-squared values (and the five percent significance value for R^2 for the sample size of 36 observations). What Figure 4 shows clearly is that, unlike most of the 1980s and 1990s when there was significant positive correlation between bond and stock returns (reflected in significant positive betas), that relationship appears to have disappeared at the turn of the century.

[Insert Figure 4]

As previously noted, the rolling regressions approach will only gradually capture time variation in conditional betas, and in this respect, the estimates derived from the Kalman filter and BEKK models for the post 2000 period are of particular interest. Both also show an obvious drop in beta, with the Kalman filter estimates also suggesting negative or zero betas similar to the results from the rolling regression approach. The BEKK model, while indicating a decline in beta, is not suggestive of a negative beta. This could reflect the effect of assumptions made in applying that approach which limit its applicability to later years of the sample. These include the modelling of how shocks simultaneously affect variances and covariances and the use of the full sample in estimating autoregressive parameters.

The results are of significance in several other regards.

First, as explained earlier, if long term bonds have non-zero betas, it is inappropriate to use the yield to maturity on such bonds as the proxy for the risk free rate (or zero beta rate of return) in the CAPM or in estimating the MRP. For years prior to the sample period used here, when bond markets were less well developed and there was less variability in long term rates, use of a long term rate as the risk free rate may have been acceptable. That is no longer the case.

Second, practical applications of the CAPM often involve estimating the MRP by using historical averages of the equity premium given by the difference between returns on the All Ordinaries Index and the yield to maturity on 10 year bonds. As noted earlier, the yield to maturity is not necessarily a good estimate of the expected return on debt, but even if it were, the results derived here cast doubt on the reliability of such historical equity premium estimates as a proxy for the MRP. The reason is as follows.

In principle, the MRP is the expected excess return on the market portfolio of risky assets (such as the augmented index developed here) over a risk free security. Even if that is constant over time, the observed changes in debt betas and the relative importance of equities to debt mean that there will be temporal variation in the equity premium measured as the excess return on equities over long term debt. For example, at the start of the 1980s the average (5 year maturity) debt beta (β_D), measured against the augmented index, was in the order of 0.35 and the equity share (w_e) around 0.55. In the latter part of the 1980s the corresponding figures were around 0.15 and 0.7 and by 2004, they were in the order of 0 and 0.9. Noting that the equity premium can be related to the MRP (as shown earlier in footnote 2) by

$$E(r_e) - (Er_D) = MRP\ [\ (1-\beta_D)/w_e]$$

the figures given above suggests that the equity premium was approximately 1.2 times the MRP at the start of the sample and in the late 1980s, while at the end of the sample it was 1.1 times MRP. Over the intervening period, significant fluctuations occurred.

Third, as also outlined earlier, non-zero betas for default free fixed income instruments arise solely from correlation between changes in required rates of return on bonds and either required returns on, or expected cash flows of, equity. Disentangling those effects is problematic, but the results presented here (of generally positive debt betas) suggest that systematic changes in required returns are important. This can be expected to also be of relevance for individual stock returns and indicates that attempts to assess individual stock betas solely by reference to correlations of individual cash flows with market cash flows are incomplete.

VI. CONCLUSIONS

This paper has estimated the systematic risk of Australian government bonds. The systematic risk of bonds increases approximately proportionately with maturity, but betas vary over time. Use of an augmented market portfolio (including bonds as well as equity) has the expected effect of increasing bond betas.

For most of the period considered, the beta estimates are positive and significantly different from zero and of economic significance in several regards. In particular, they indicate that it is inappropriate to use a long term government bond rate as an estimate of the risk free interest rate in the CAPM (where short term holding periods are assumed) because long term bonds have not been zero beta assets. Second, the results indicate that in estimating the weighted average cost of capital and its components, analysts and firms may need to consider the effect of a non zero systematic risk of debt. Third, the results indicate that systematic changes in required rates of return are not unimportant, implying that individual equity betas are likely to be determined at least partly by such effects as well as by correlation between a firm's expected cash flows and market conditions. Finally, the existence of systematic risk in bond portfolios has implications for optimal portfolio formation and hedging activities of portfolio managers.

Extensions to the work conducted here could occur on at least four fronts. First, further work could be done in refining and extending the measurement and use of a broader market portfolio. Estimation of systematic risk of both equities and bonds using a broader portfolio of risky assets which also includes State Government and corporate bonds and asset backed securities would be appropriate.[18] Second, further work could be undertaken examining the impact of assuming longer holding periods (such as three or six months) on beta estimates for bonds. Third, alternative time series techniques could be used to investigate further the precise nature of time variation in beta estimates. Finally, use of a multi-factor model for asset returns (rather than the simple one-factor CAPM model applied here) could also be considered.

Finally, a particularly important contribution of this paper in terms of future research lies in the challenge the results pose to researchers to explain the apparent de-linkage of equity and long term bond markets experienced at the start of the millennium and reflected in the marked decline of bond betas.

Table I
Australian Government Benchmark Bonds

Maturity	Amount Issued (up till Dec 2003)	Coupon	First Issued
15 Mar 1994	1,998	10.00	3-Sep-1991
15 Sep 1994	1,906	12.50	28-Aug-1984
15 Apr 1995	1,852	12.50	25-Sep-1984
15 Sep 1995	3,399	10.50	9-Jul-1991
15 Jul 1996	2,581	13.00	13-Jan-1983
15 Mar 1997	1,797	12.5	14-Oct-1986
15 Sep 1997	2,797	12.5	28-Aug-1984
15 Jan 1998	3,203	12.5	18-Aug-1987
15 Aug 1998	4,589	7	4-Aug-1992
15 Mar 1999	4,639	6.25	13-Mar-1984
15 Jul 1999	2,704	12	5-Jul-1988
15 Apr 2000	5,795	7	15-Apr-1993
15 Jul 2000	2,047	13	9-Jul-1985
15 Jan 2001	5,099	8.75	20-Jul-1994
15 Nov 2001	2,253	12	20-May-1986
15 Mar 2002	3,706	9.75	5-Apr-1995
15 Oct 2002	4,097	10	8-Oct-1991
15 Aug 2003	4,789	9.5	14-Apr-1992
15 Sep 2004	5,511	9	19-Nov-1992
15 Jul 2005	5,202	7.5	15-Jul-1993
15 Feb 2006	3,903	10	22-Feb-1995
15 Nov 2006	5,902	6.75	18-Nov-1993
15 Oct 2007	4,007	10	23-Nov-1994
15 Aug 2008	4,294	8.75	12-Sep-1995
15 Sep 2009	5,509	7.5	29-Oct-1996
15 Jun 2011	4,695	5.75	25-Aug-1998
15 May 2013	4,300	6.5	23-May-2000
15 Apr 2015	3,897	6.25	28-May-2002

Source http://www.rba.gov.au/Statistics/Bulletin/E05Ahist.xls

Table II

Fixed Interest Betas

This table presents output from OLS regression equations of the form:

$$r_n - r_f = \beta_0 + \beta_1 (r_m - r_f) + u_i$$

where r_n is the one month holding period return on a government security of maturity n, r_f is the risk free (one month Bank Accepted Bill) rate and r_m is the return on a market portfolio.

Panel A presents results using the All Ordinaries Accumulation Index as the market portfolio. Panel B presents results using a broader market portfolio which also includes government bonds. All returns were calculated as continuously compounded returns per month. The data period was December 1979 to February 2004 (291 observations). (t statistics are in are in parentheses).

Panel A: Betas using an equity market index

		Maturity	
	2 Year	**5 Year**	**10 Year**
β_0	-0.0008	0.005	0.011
	(-0.14)	(0.45)	(0.67)
β_1	0.046	0.10	0.15
	(5.39)	(5.82)	(5.88)
R^2	0.09	0.09	0.10
Durbin-Watson	1.69	1.69	1.67

Panel B: Betas using an augmented (equity plus debt) market index

β_0	-0.0008	0.005	0.011
	(-0.15)	(0.47)	(0.71)
β_1	0.08	0.18	0.25
	(7.37)	(8.14)	(7.98)
R^2	0.15	0.19	0.18
Durbin-Watson	1.68	1.67	1.66

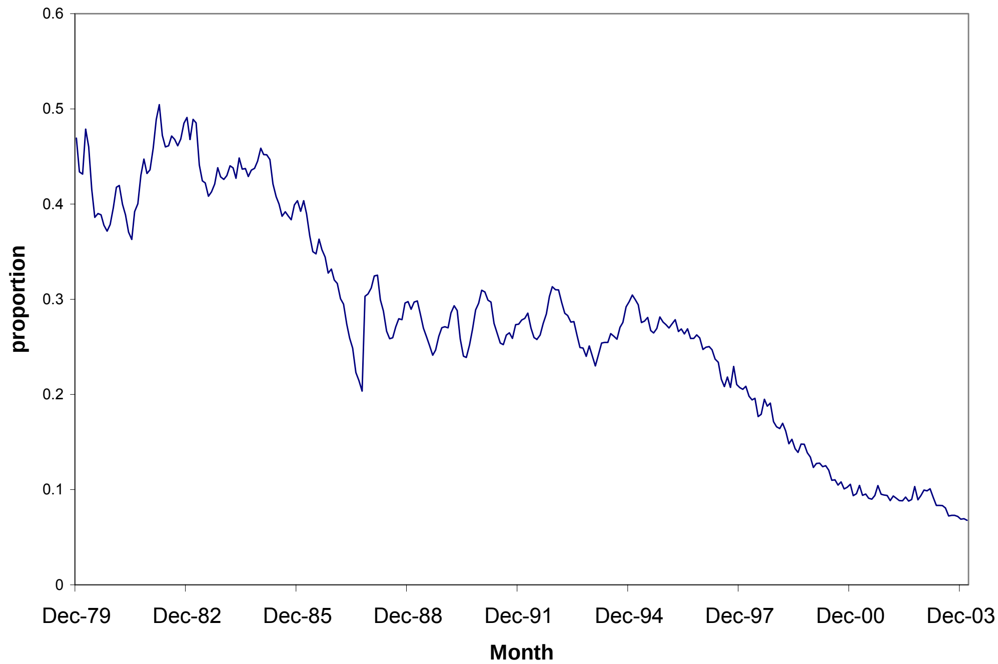

Figure 1
Market Capitalization Share:
Bonds/(Equities + Bonds)

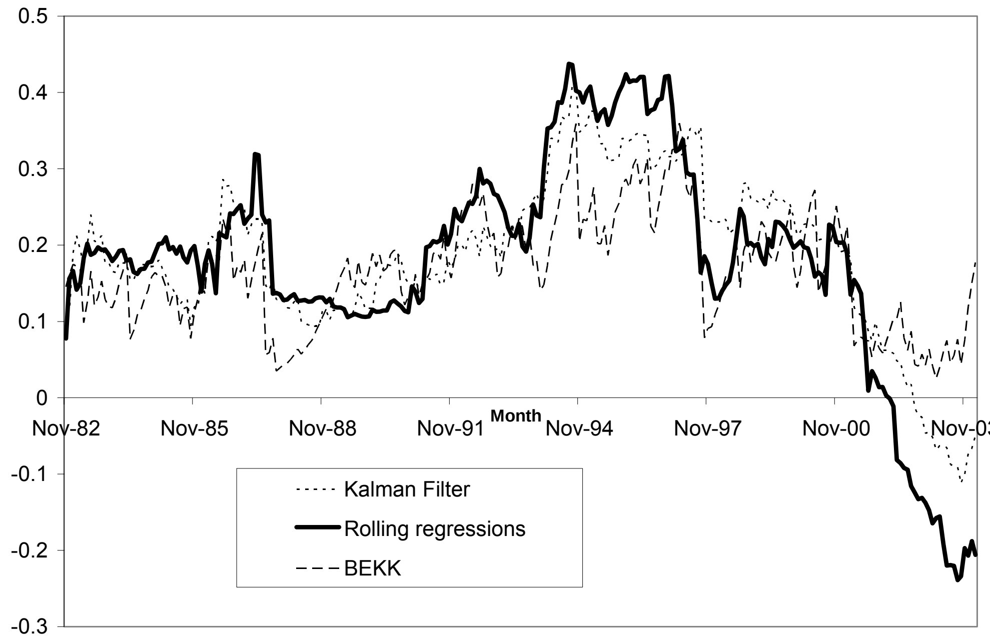

Figure 2
10 Year Bond Betas: Alternative Estimators

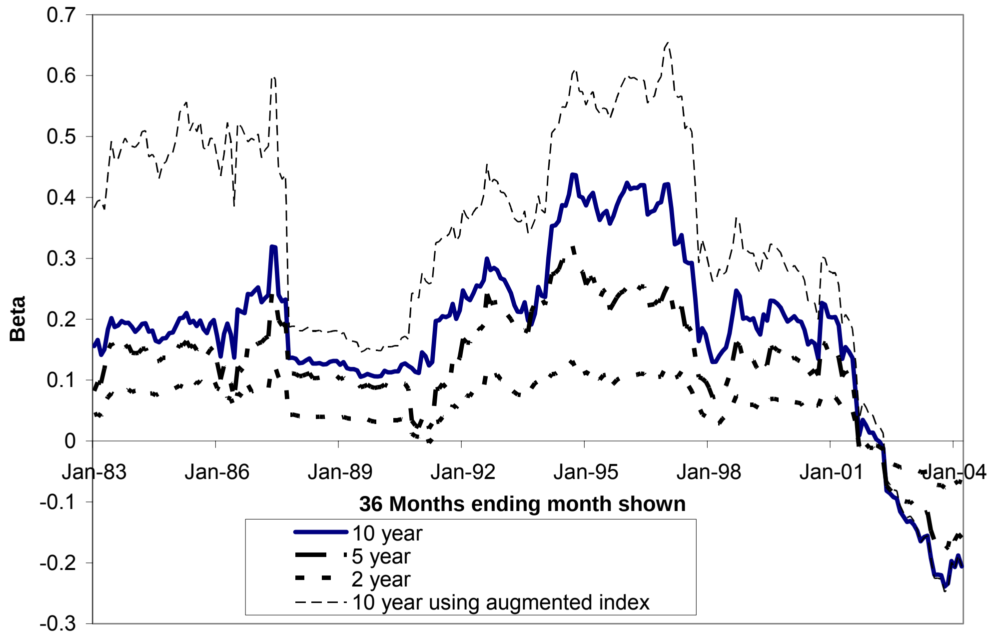

Figure 3
Bond Betas: Rolling 36 Month regressions

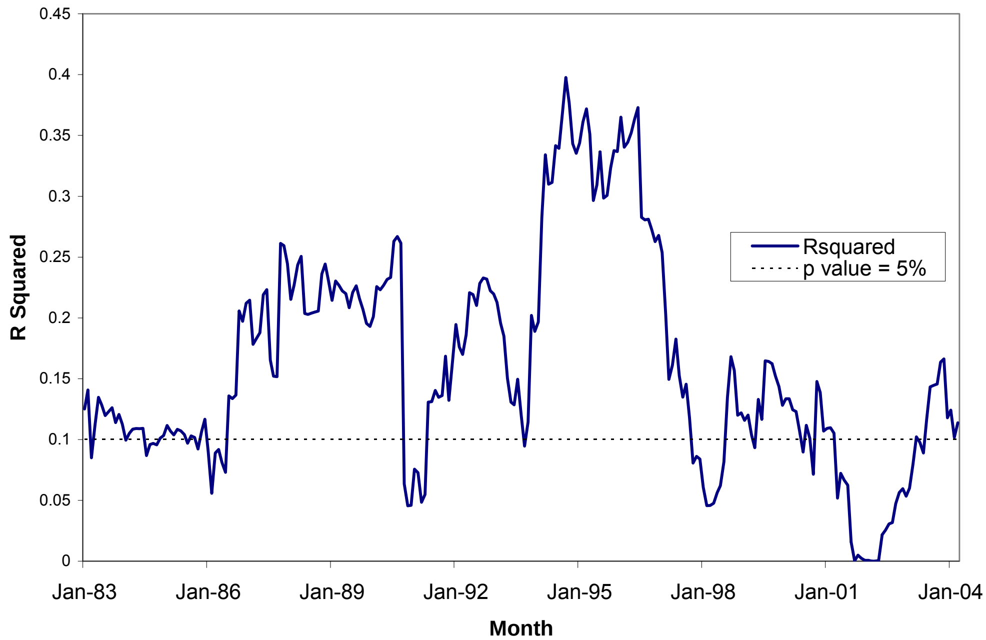

Figure 4
10 Year Bond Betas: R squared of Rolling 36 Month regressions
$$(R_{10} - R_f)_t = \beta_0 + \beta_1(R_m - R_f)_t + u_t$$

APPENDIX 1

Constructing holding period yields from Actual yield data

Data was provided by the Reserve Bank of Australia in the form of daily end of day indicative mid rates[19] for Commonwealth Government bonds for the period July 1992 to December 2003. The Australian market convention is that bonds are traded on the basis of yield to maturity, with the settlement price calculated as a "dirty price" (inclusive of accrued interest) using the Reserve Bank bond pricing formula. Interest is paid semi-annually (on the 15th of the month) using an "actual/ 365 days" calculation.

Monthly holding period returns for hypothetical bonds of two, three, five, and ten year maturities were calculated for the period July 1992 to December 2003 using the following method. First, the daily data for benchmark bonds was filtered to extract end of month observations. Second, the price of benchmark bonds at the end of each month was calculated using the Reserve Bank pricing formula[20]. Third, monthly holding period returns for the month ending at date t+1 ($r_{t,t+1}$) were calculated as a continuously compounded rate of return (per month) using:

$$r_{t,t+1} = ln(\frac{P_{t+1}}{P_t})$$

for months in which there was no coupon interest payment and

$$r_{t,t+1} = ln(\frac{P_{t+1} + C}{P_t})$$

for months in which there was a coupon payment of amount C on the 15th of the month[21].

To derive monthly holding period returns for hypothetical bonds of, for example, five years term to maturity the following interpolation process was used. First, at each point in time, the pair of bonds with nearest maturities spanning five years was selected. The holding period return for the five year bond was calculated as a weighted average of the holding period return for those two bonds according to the formula:

$$r^{5}_{t,t+1} = r^{L}_{t,t+1}\left(\frac{5-L}{M-L}\right) + r^{M}_{t,t+1}\left(\frac{M-5}{M-L}\right)$$

where M (L) refers to the term to maturity of the bond with greater (lesser) maturity than five years in the spanning pair.

Holding period returns were calculated for hypothetical bonds of 2, 3, 5, and 10 year maturities.

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ENDNOTES

[1] Faff, Brooks and Ho (2002) discuss problems with leverage adjustments which incorrectly assume a zero debt beta.

[2] Note that the beta of the market portfolio must equal 1, and aggregating equities and bonds into two asset classes, $\beta_e w_e + \beta_D(1-w_d) = 1$ where β_e and β_D are the beta of equity of equity and bonds (relative to the aggregate market portfolio of equities and bonds) and w_e is the weight of equities in that aggregate portfolio. Thus $E(r_e) – (Er_D) = MRP [(1-\beta_D)/w_e]$ such that $E(r_e) – E(r_D)$ will vary inversely with β_D and w_e.

[3] More precisely, the condition should be stated as duration being equal to the holding period assumed since equality of duration and holding period makes the security's rate of return insensitive to small interest rate changes. For ease of exposition, and because most holding periods considered are sufficiently short that the relevant debt instruments are discount securities (for which duration equals maturity) this complication is ignored.

[4] Amato and Remolona (2003) review some of the international evidence on what they term the "credit spread puzzle" and argue that one reason for large credit spreads (after adjusting for tax and liquidity effects) is that the credit risk involved is, in part, non-diversifiable. This indicates a role for other factors in addition to the market return in determination of expected returns.

[5] Dichev (1998) concludes from a study of US equity returns that bankruptcy risk is not a systematic risk and thus not priced in equity returns.

[6] Modified duration is a statistic which summarises the temporal distribution of cash flows of a fixed income bond. It also provides an approximation for the sensitivity of a bond's price to small changes in its yield to maturity. See Fabozzi (1996, p 60) for more detail.

[7] Jarrow (1978) derives a similar relation assuming that a bond's yield to maturity and stock prices follow Geometric Brownian Motion with drift.

[8] All data used was sourced from the Reserve Bank web site.

[9] The beta of any given bond with a fixed maturity date will vary over time as its time to maturity and duration shorten. See Chen (1989).

[10] It is also worth noting the confirmatory evidence that the correlation between the two time series of ten year bond betas estimated from 36 month rolling regressions using the two alternative approaches was in excess of 0.99.

[11] Bank Bills and Treasury Notes are discount instruments priced according to the formula $P_t = F/(1+y_{t,t+h}h/365)$ where F is the face value to be received at date t+h.

[12] The average maturity varied between 56 and 75 months for the sample period.

[13] Estimation of a "market model" $r_n = \beta_0 + \beta_1 r_m$ (which assumes no change in the risk free rate over time and effectively incorporates any such changes into the constant term) gave virtually identical results.

[14] The Durbin Watson statistics reported in Table 2 are in the indeterminate region, but suggestive of positive autocorrelation. The equations were re-estimated assuming a first order autoregressive structure for the error term. The beta estimates and t statistics were essentially unchanged.

[15] Such covariance could arise from changing investor expectations about equity cash flows leading to substitution between equity and debt, or from monetary policy adjustments to interest rates in response to perceived changes in investor optimism or pessimism.

[16] The difference between the arithmetic means is 0.25 per cent p.a.

[17] For this reason, many financial market analysts use exponential weighting of observations in calculating volatility to give greater weight to recent observations and more rapid adjustment of estimates to contemporaneous events.

[18] While the relative importance of Commonwealth Government Bonds has declined, the relative importance of State Government and corporate and asset backed bond markets has increased.

[19] Average of buy/sell rates reported by bond dealers surveyed by the Bank at 4.30 pm AEST.

[20] Because bonds pay interest on the 15th of each month and go ex interest 14 days prior to that, all end of month observations are of *cum-interest* bonds.

[21] The error in holding period returns induced by the implicit assumption in this formula that the coupon is received at the end of the month is minor.